Advance sales and deterrence with heterogeneous firms

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Abstract

We examine the effects of firm heterogeneity when firms can compete in advance for future demand by either entering forward contracts or by selling to agents that store the good to meet future demand. Firms' sales in the second period are reduced by aggregate advance sales, so high-cost firms may produce zero output in equilibrium if aggregate advance sales induce a price below their marginal cost. The endogenous number of active firms leads to the possibility of a deterrence equilibrium in which lower-cost firms act to deter the activity of higher-cost firms. In this case, the presence of inactive higher-cost firms in the market results in a lower price than would otherwise obtain. In addition, the advance sales equilibrium with heterogeneous firms has higher market shares for relatively efficient firms compared to that in both the heterogeneous firm Cournot equilibrium and the homogeneous firm advance sales equilibrium. Consequently, the equilibrium outcome results in industry output produced at a lower average cost, which represents an additional welfare gain associated with the pro-competitive effects of strategic advance sales even though the reallocation of market shares leads to higher measured concentration.

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1 Introduction

In many industries the time at which a firm sells its product and the time at which that product is consumed need not be the same, allowing firms to be able to make sales for a particular demand in multiple periods. There are two common situations in which this occurs. First, firms can sell in advance via contracts to supply the good on a future date and then can supply additional quantities via the spot market on that future date. Second, for storable goods, sales that firms make in one period can be used to meet demand in a future period, either because consumers store the good for their own future consumption, or because intermediaries store the good for future resale. If firms are imperfectly competitive, this ability to sequentially supply a future demand introduces the possibility that advance sales may be used to manipulate future spot market competition. For example, under quantity competition advance sales allow firms to preemptively capture a portion of future demand, which can lead to more aggressive competition. A second common feature of many industries is the co-existence of firms operating with different production technologies, leading to heterogeneous costs of production even if the products or services they sell are homogeneous. For example, mineral extraction firms operate mines with different ore grades, and electricity generators operate plants with different fuel inputs and efficient ratings. Although the effect of advance sales has been extensively studied in the context of homogeneous firms, the more general case of heterogeneous firms has not seen much attention. By affecting the competitiveness of the equilibrium, advance sales with heterogeneous firms may also affect the number of firms that make positive sales in equilibrium. In particular, they raise the possibility that relatively efficient firms may use advance sales to deter less efficient rivals.

We analyze a simple oligopoly model in which a set of firms that differ in their marginal cost can sell both in advance and spot. The number of active firms (those with positive sales) is endogenous as higher cost firms may find the equilibrium price at or below their marginal cost due to advance sales by their more efficient rivals. The equilibrium price and aggregate sales are unique, however, the equilibrium can be one of two general types, depending on whether an inactive firm is blockaded or deterred.[1] When all inactive firms are blockaded, the equilibrium is qualitatively similar to that in a homogeneous firm model with the same number of firms and average marginal cost, however, market shares are skewed towards relatively efficient firms. In a deterrence equilibrium, efficient firms non-cooperatively choose a level of aggregate advance sales that causes the price to equal the marginal cost of the deterred firm. Hence, deterrence occurs even though there are no fixed costs in our model. Although there are a continuum of equilibria in this case, they all have the same equilibrium price and aggregate sales, differing only with the allocation of the unique aggregate advance sales among the individual firms. With deterrence, price is lower than that in a comparable homogeneous firm model with the same number of active firms. Consequently, firm heterogeneity can enhance the pro-competitive effects of advance sales, although it need not do so because fewer firms may be active in equilibrium than in the homogeneous firm benchmark.

The equilibrium distribution of production across firms represents an additional av-

[1]We borrow the terms blockaded, deterred, and accommodated from the entry deterrence literature, however there is no entry in our model. Instead, we use these terms to refer to firms "activity" in the market and the three terms coincide with price being below, equal to, or above a firm's marginal cost respectively.

enue by which efficiency is affected by advance sales. Compared to the heterogeneous firm Cournot benchmark, advance sales result in a reallocation of market share towards relatively low-cost firms, so total output is produced at a lower average cost. Compared to a comparable homogeneous firm benchmark with advance sales, even in situations in which total output and price are the same, the average cost of total industry production is lower due to the higher share of production accounted for by low cost firms. Consequently, in addition to the pro-competitive effects of advance sales that has been established in the case of homogeneous firms, the reduction in industry average cost is another avenue by which welfare may be improved by advance sales.

There has been substantial interest in the strategic effects of advance sales in the form of forward contracts since Allaz and Vila (1993) demonstrated that increasing the number of forward trading opportunities causes an increasingly competitive outcome. This pro-competitive effect of forward trading was questioned in a number of subsequent articles. Liski and Montero (2006) and Mahenc and Salanié (2004) demonstrate that competition can instead be reduced by forward trading, whereas Adilov (2012) shows that if firms choose their production capacity prior to the forward contracting decisions, the Cournot equilibrium re-emerges as the unique outcome. Strategic contracting has been of particular interest in the analysis of electricity markets, where forward contracting plays a particularly significant role. Holmberg and Willems (2015) and Bushnell (2007) are a couple of examples of extensions of the Allaz and Vila (1993) framework to relevant aspects of trading in electricity markets. More generally, many commodity markets share this feature of a longer term contracting market coexisting with a short-term/spot market in the presence of imperfect competition, however, not much attention has been paid to role of firm heterogeneity, which is another common characteristic of these markets. There are a couple of exceptions to this observation. In the electricity market context de Frutos and Fabra (2012) consider heterogeneous firms that compete in supply functions, however, forward commitments are exogenously chosen by a regulator in their model. Miller and Podwol (2020) allow for firm heterogeneity in a model of quantity competition with forward contracting in order to analyze mergers in this setting. Firms differ only in the slopes of their increasing marginal cost functions, so in their model deterrence is not possible, and all firms are active in equilibrium. In contrast, firms in our model have constant marginal cost that differs across firms, so it is possible for price to be above the marginal cost of some firms and below the marginal cost of others, resulting in the possibility that some firms are inactive in equilibrium.

Another type of advance sale occurs when the good is storable:[2] if not consumed when purchased, the good remains available and "competes" with firms' future production. This introduces a dynamic element to demand when consumers can store the good, which has been shown by Hendel and Nevo (2006) to have significant implications for demand estimation. Theoretical analyses of consumer storage include Dudine et al. (2006) and Antoniou and Fiocco (2019) who examine the case of a monopolist seller, and Anton and Das Varma (2005) and Guo and Villas-Boas (2007) who examine duopoly sellers. The closest of these to our model is Anton and Das Varma (2005), who examine a homogeneous firm duopoly and show that in equilibria where storage occurs, strategic behaviour leads to higher output and lower prices than is the case in the absence of consumer storage. A similar effect occurs when the good is storable by other competitive intermediaries instead of consumers. Mitraille and Thille (2009) analyze the effects of

[2]This is noted also in Mitraille and Thille (2020)

speculative storage on a monopoly's dynamic behaviour, and Mitraille and Thille (2014) consider the effect of speculative storage in a multi-stage oligopoly. Similarly to the case with consumer storage, speculative storage, when positive, transfers demand from a future period to the present, resulting in a strategic incentive to increase sales. In this case, the resale of speculative inventories represents additional supply in the future period when speculators sell their stocks. Whether storage is undertaken by consumers or speculators, the excess of purchases over consumption in an earlier period is essentially advance sales since producers are selling the good that will be consumed in a future period. The effect on future demand is the same as it is under the contracting interpretation of advance sales.

Our contribution to this advance sales literature is to demonstrate that firm heterogeneity can change the qualitative nature of the equilibrium due to the endogeneity of the number of active firms. In particular, lower-cost firms may act even more competitively in order to deter the activity of higher-cost rivals. In addition, we show that the equilibrium with heterogeneous firms can result in the reallocation of market share from high-cost producers to low-cost producers, which represents an additional channel of potential welfare improvement due to advance sales that is absent in the homogeneous firm setting.

Finally, our results have bearing on a couple of topics in competition policy. The relationship between measured concentration and market power is quite different compared to a static model: the HHI is larger and the Lerner index is lower than what would obtain in the both static heterogeneous Cournot oligopoly case, and in the homogeneous advance sales case. In addition, merger analysis is complicated by the possibility of advance sales as the profitability of a merger and its effect on both price and industry average cost depend significantly on the nature of both the pre- and post-merger equilibria.

We present our model in the next section and follow that with our results regarding the conditions under which deterrence equilibria occur. We then break up our analysis of the equilibria into two parts, first examining the effect of heterogeneity on the competitiveness of the equilibrium, and then examining the implications for the equilibrium allocation of advance sales across individual firms. In the penultimate section we consider implications for some aspects of competition policy, and then conclude with a discussion of the potential robustness of our results.

2 Model

There are N producers that may sell their output in two sequential markets: a market for advance sales followed by a market for spot sales. In the advance market each producer $i{=}1, 2, ...N$ chooses a quantity x_i to sell to competitive agents at a price p_A. These advance sales can take the form of forward contracts specifying the delivery in the second period, or of physical sales to consumers or speculators, who store the product for use in the second period.[3] In either interpretation, advance sales reduce the net demand faced by producers in the spot market. We define aggregate advance sales as $X{=}\sum_{i=1}^{N} x_i$, and the aggregate sales of i's rivals as $X_{-i}{=}\sum_{j\neq i} x_j$. The vector of advance sales, $(x_1, x_2, ..., x_N)$, and the advance price, p_A, are observable to all firms at the beginning of the second

[3]In this case, we assume zero storage costs for any agent storing the good.

period.[4] In the second period spot market each producer may choose to produce an additional quantity y_i to sell on the spot market at a price p_S, with $Y{=}\sum_{i=1}^{N} y_i$ and $Y_{-i}{=}\sum_{j\neq i} y_j$.

A firm's total production, q_i, must fulfill the firm's combined advance and spot sales, $q_i{=}x_i{+}y_i$, which is produced at a cost of $C_i(q_i){=}c_iq_i$. Constant marginal cost and zero fixed costs are important for the analysis as they imply that firms will not have a cost incentive to produce all output in a single period. In addition, zero fixed costs are important as they would provide a reason for high-cost firms to choose zero production in addition to that on which we wish to focus, which is high-cost firms choosing zero production in response to a non-positive price-cost margin instead of a non-positive profit. We index firms in increasing order of their marginal costs of production with firm one being the most efficient:[5] $0{\leq}c_1{<}c_2{<}...{<}c_N$. Let $\bar{c}_N{=}\frac{1}{N}\sum_{i=1}^{N} c_i$ denote the average marginal cost across all firms, and $\bar{c}_k{=}\frac{1}{k}\sum_{i=1}^{k} c_i$ for $k{<}N$ denote the average cost of the k lowest-cost firms. The vector of marginal costs, $\boldsymbol{c}{=}(c_1, c_2, ..., c_N)$ is known to all firms.

Since marginal cost differs across firms, we must allow for the possibility that firms with high marginal cost may not find it profitable to sell positive quantities in equilibrium. We say a firm is *active* in a market if it makes positive sales in that market. So a firm is active on the spot market if $y_i{>}0$ and active on the advance market if $x_i{>}0$, and is *inactive* when sales are zero.

In order keep the alternative interpretations of the model consistent with our analysis, we assume that the timing of the accrual of revenues and costs is irrelevant to the firms' payoffs, i.e., firms have a common discount factor of one.[6] The total profit for a producer from both advance and spot sales is then[7]

$$\pi_i{=}(p_A{-}c_i)x_i{+}(p_S{-}c_i)y_i. \tag{1}$$

There is a single demand for the product by consumers that can be satisfied by either advance or spot purchases. This is clearly the case in most two-period models of strategic forward sales, but for models of sales to consumers/intermediaries who store, there is a spot market operating in the first period as well where these agents purchase their inventories, and hence, a separate demand for the good in the first period. In interpreting our model in this latter case, advance sales are simply the excess of what is purchased versus consumed in the first period and p_A is the first period spot price. Note

[4]The assumption that firms observe the entire vector of advance sales is stronger than necessary. See Ferreira (2006) for an extended treatment of this issue in the forward contracting setting of Allaz and Vila (1993). All that is required is that firm i knows x_i and X, the latter of which can be inferred from p_A if required, so at a minimum firms only need to know their own advance sales and the advance price in our model.

[5]Although the analysis could be done allowing more than one firm to have the same marginal cost, the presentation of the results is substantially simplified by imposing that firms have distinct marginal costs.

[6]Our results would not be significantly affected by a positive rate of discount, however the presentation and interpretation of the results would be more complicated. In particular, there would be slight differences if the advances sales were due to contracting, with all revenue and costs occurring in the second period, versus prior sales to consumers or intermediaries, with the revenue and costs of advance sales occurring prior to those of spot sales.

[7]This profit highlights the difference between advance sales and advance *production* (Saloner (1987), Pal (1991, 1996), and others), in which firms commit to a minimum level of sales. However, in those models committed sales are sold on the spot market at the spot price, so profit from committed production is not determined in the first period, unlike in the advance sales models.

that we are implicitly assuming that in this case we are not in the situation in which first period demand/price is so high that no agents wish to store for the second period, i.e. we do not consider the possibility that first period demand for advance sales is zero, as can occur in Anton and Das Varma (2005) and Mitraille and Thille (2014). We assume a linear inverse demand, $P(Q)=a-Q$. When advance sales are purchased by intermediaries who store in order to resell in the spot market, advance sales represent an alternative source of supply in the spot market, so total supply in the spot market is $Q=X+Y$. When advance sales are purchased by consumers and stored, demand in the spot market is reduced by advance sales:[8] $P=(a-X)-Y$. In either case, given total advance sales, X, inverse demand in the spot market is then

$$p_S=P(X+Y)=a-X-Y. \tag{2}$$

We make no particular assumptions regarding the parameter a beyond $a>c_1$, i.e., demand is large enough that at least the most efficient firm will be active. Since we wish to examine comparative statics compared across different market structures and the competitiveness of the equilibrium will in principle vary across these market structures we wish to allow flexibility in the set of firms that are active in equilibrium, and so do not make any assumption regarding the demand parameters that pre-judges the number of active firms.

Demand for advance sales is derived from the equilibrium spot price, p_S. The per-unit expected profit to an advance purchase is p_S-p_A, which is driven to zero by the competitive agents purchasing in the advance market.[9] Given the absence of uncertainty and perfect foresight among market participants, the inverse demand for advance sales, $p_A(X)$, will equal the equilibrium spot price that obtains when aggregate sales are X.

A strategy for a firm consists of its choice of advance sales, x_i, and its choice of spot sales in each sub-game. Sub-games differ with respect to the vector of advance sales for each firm. However, as we see from (1), the profit from spot sales depends on advance sales only through their effect on the spot price. Since the spot price is a function of aggregate spot sales only, the distribution of advance sales across individual firms does not affect a firm's choice of spot sales. Consequently, we write $y_i(X)$ to denote firm i's spot sales strategy. The payoff for any firm $i=1,...,N$ given strategies for each producer $\{x_j, y_j()\}$, $j=1,...,N$, is then

$$\pi_i=\left(a-X-\sum_{j=1}^{N} y_j(X)\right)(x_i+y_i(X))-c_i\left(x_i+y_i(X)\right). \tag{3}$$

We analyze the sub-game-perfect equilibrium of this game.

[8] If we think of demand as generated by heterogeneous consumers that differ in their willingness to pay for the good, we are assuming here that advance purchases are made by the consumers with the highest valuations. Anton and Das Varma (2005) demonstrate that the choice of rationing rule does not affect the results.

[9] This is just a requirement that there be no arbitrage opportunities and is the standard condition in models of storage with competitive agents as well as in strategic forward trading models. See Ito and Reguant (2016) for an example in which arbitrage is imperfect.

3 Equilibrium

3.1 Equilibrium in spot market sub-games

The equilibrium in the spot market depends on the aggregate advance sales, X, as this determines the net demand that is faced by firms. Since firms differ in their marginal costs, the set of firms that are active in the spot market depends on the aggregate advance sales, since higher-cost firms will not sell if X is sufficiently large. Consequently, the number of firms active on the spot market, which we denote $n(X) \leq N$, will be determined by comparing the level of advance sales, X, to thresholds that determine whether a given firm wishes to sell in the spot market.

Spot market sub-games are defined by the aggregate level of advance sales, X, in which each firm solves

$$\max_{y_i} \{(a-(X+Y)-c_i)y_i+(p_A-c_i)x_i\} \qquad i=1, ..., N. \tag{4}$$

The aggregate advance sales, X, reduces the level demand in that sub-game, so each sub-game is simply akin to a Cournot game with inverse demand of $p_S=(a-X)-Y$, i.e., aggregate advance sales effectively reduces the intercept of the inverse demand curve.[10] The marginal profit of spot sales is

$$\frac{\partial \pi_i}{\partial y_i}=a-X-Y_{-i}-c_i-2y_i. \tag{5}$$

Suppose that all N firms are active, the equilibrium individual spot sales would be the same as in the heterogeneous-firm Cournot game with inverse demand intercept of $a-X$:

$$y_i^*=\frac{(a-X)+\sum_{j=1}^{N-1} c_j-Nc_i}{N+1}=\frac{a-X+N\bar{c}_N}{N+1}-c_i \qquad i=1, 2, ..., N, \tag{6}$$

and

$$p_S^*=\frac{a-X+\sum_{j=1}^{N} c_j}{N+1}=\frac{a-X+N\bar{c}_N}{N+1}. \tag{7}$$

For all N firms to be active, price must exceed the marginal cost of the least efficient firm,[11] which requires $a-X>(N+1)c_N-N\bar{c}_N$. For lower values of $a-X$, firm N is not active. Similarly, if a sub-game equilibrium has $k<N$ firms active, we have

$$y_i^*=\frac{a-X+\sum_{j=1}^{k} c_j-(k+1)c_i}{k+1}=\frac{a-X+k\bar{c}_k}{k+1}-c_i \qquad i=1, 2, ..., k, \tag{8}$$

and

$$p_S^*=\frac{a-X+\sum_{j=1}^{k} c_j}{k+1}=\frac{a-X+k\bar{c}_k}{k+1}. \tag{9}$$

Price must exceed c_k for k to be active, or $a-X>(k+1)c_k-k\bar{c}_k$. However, for this to be an equilibrium, it must also be the case that the price is lower than firm $k+1$'s marginal cost, i.e., $a-X<(k+2)c_{k+1}-(k+1)\bar{c}_{k+1}$. Consequently, we can define threshold levels of aggregate advance sales that determine whether a firm is active or inactive on the spot market:

[10] In a setting where demand is uncertain, such as Mitraille and Thille (2020), the possibility that $X>a$ needs to be considered as it may occur on the equilibrium path. However, a firm choosing $x_i>0$ when $X_{-i}>a$ is not rational in our setting, so $X>a$ can be ignored.

[11] Alternatively, $y_N^*>0$, which gives the same condition.

Definition 1. *Firm k is active on the spot market if $X{<}X_k^d$ and inactive on the spot market if $X{\geq}X_k^d$ where*

$$X_k^d{=}a{+}k\bar{c}_k{-}(k{+}1)c_k. \tag{10}$$

It is straightforward to show that these activity thresholds form a decreasing sequence: $a{-}c_1{=}X_1^d{>}X_2^d{>}X_3^d{>}...{>}X_N^d$.[12] The number of firms active on the spot market when aggregate advance sales are X is then given by the number of firms for which $X{<}X_k^d$. In summary, we have

Proposition 1. *In a sub-game with aggregate advance sales of X, the number of active firms is given by*

$$n(X){=}\sum_{j=1}^{N}\mathbb{1}_{[X<X_k^d]}, \tag{11}$$

equilibrium spot sales strategies are the continuous functions

$$y_i^*(X){=}\max\left[\frac{a{-}X{+}n(X)\bar{c}_{n(X)}}{n(X){+}1}{-}c_i, 0\right] \qquad \forall i, \tag{12}$$

and the spot market equilibrium price is the continuous function

$$p_S^*(X){=}\frac{a{-}X{+}n(X)\bar{c}_{n(X)}}{n(X){+}1}. \tag{13}$$

Proof. See Appendix A.

Notice that the spot market sub-game with $X{=}0$ is simply the Cournot game with heterogeneous firms, so Proposition 1 also provides the Cournot equilibrium in this case, where a firm will be active in the Cournot equilibrium if $X_k^d{>}0$. Since the X_k^d vary with a and $\boldsymbol{c}$, the number of firms active in the Cournot equilibrium will also not necessarily equal N. We will be comparing the outcome of the advance sales game to that of the Cournot game below, so, for completeness, we state the Cournot equilibrium as a Corollary to Proposition 1:

Corollary 1. *The equilibrium in the Cournot game has n_C active firms, with $n_C{=}n(0){\leq}N$, individual output $y_i^C{=}y_i^*(0)$, $i{=}1, 2, ...N$, and price $p^C{=}p_S^*(0)$.*

3.2 Equilibrium advance sales

Having determined the equilibrium in each spot market sub-game, all that is left for determining a firms profit as a function of advance sales is to specify the equilibrium advance sales price, p_A. Given an aggregate quantity of advance sales, X, the equilibrium price for advance sales is driven by the competitive purchasers of the advance sales to equal the equilibrium spot price:

$$p_A^*{=}p_S^*(X){=}a{-}\left(X{+}\sum_{i=1}^{n(X)}y_i^*(X)\right), \tag{14}$$

[12] To see this, express $X_k^d{>}X_{k+1}^d$ as $c_{k+1}{+}(k{+}1)(c_{k+1}{-}c_k){>}(k{+}1)\bar{c}_{k+1}{-}k\bar{c}_k$. As $(k{+}1)\bar{c}_{k+1}{-}k\bar{c}_k{=}c_{k+1}$, this reduces to $c_{k+1}{>}c_k$ which is true by our assumption regarding firms' marginal costs.

which represents the inverse demand faced by producers for their advance sales when their spot market behaviour is correctly anticipated by all market participants.

Having determined p_A^*, and given the spot market equilibrium defined in Proposition 1, the profit faced by a firm when choosing its advance sales in the first period is

$$\pi_i(x_i, X_{-i}) = (p_S^*(x_i + X_{-i}) - c_i)(y_i^*(x_i + X_{-i}) + x_i). \tag{15}$$

Both the spot sales strategies, $y_i^*(X)$, and the spot price, $p_S^*(X)$, are continuous, but kinked functions of aggregate advance sales. Consequently, the firm's profit is a continuous, kinked function of its advance sales, with marginal profit discontinuous where the level of advance sales induces a higher-cost firm to become active/inactive, i.e., where $x_i + X_{-i} = X_k^d$.

Using Proposition 1 we can write a firm's reduced form profit as

$$\pi_i(x_i, X_{-i}) = \left(\frac{a - X + n(X)\bar{c}_{n(X)}}{n(X)+1} - c_i\right)\left(x_i + \max\left[0, \frac{a - X + n(X)\bar{c}_{n(X)}}{n(X)+1} - c_i\right]\right), \tag{16}$$

with $X = x_i + X_{-i}$. There are two sources of kinks in firm i's profit, and hence discontinuities in i's marginal profit: first, when firm i is indifferent between being active or inactive in the spot market, and, second, when firm $j > i$ is indifferent between being active or inactive in the spot market. The latter is due to the discontinuous nature of $n(X)$ as the number of firms changes when the activation thresholds of less efficient firms are reached. The first term in (16) is simply $p_S^*(X) - c_i$, so if firm i is inactive on the spot market, $(p_S^*(X) < c_i)$, it is also inactive on the advance market as profit is negative for any positive level of advance sales. This result means that we can ignore the kink at firm i's activation threshold (due to the max operator in (16)) when determining the firm's optimal advance sales since they will not be positive when the firm is not active on the spot market.

Marginal profit for active firms is then

$$\frac{\partial \pi_i(x_i, X_{-i})}{\partial x_i} = \left(\frac{a - X + n(X)\bar{c}_{n(X)}}{n(X)+1} - c_i\right)\frac{n(X)-1}{n(X)+1} - \frac{x_i}{n(X)+1}, \qquad \forall i \leq n(X), \tag{17}$$

which is discontinuous at the activity thresholds for each firm $k > i$ where there is a discrete change in $n(X)$.[13] Clearly, marginal profit for firm i is linear and decreasing apart from the points where $x_i + X_{-i} = X_k^d$, for $k > i$. From Proposition 1 we know that firm k will be active if $X < X_k^d$, which we can write from firm i's point for view as $x_i < X_k^d - X_{-i}$. The following Lemma establishes that marginal profit only jumps downward at these discontinuities:

Lemma 1. *Given X_{-i}, marginal profit for firm i is strictly decreasing in x_i with downward jump discontinuities at values $x_i = X_k^d - X_{-i}$ for $k > i$ and $X_{-i} < X_k^d$.*

Proof: See Appendix A.

From Lemma 1, the firm's profit maximizing response to X_{-i} will be one of two qualitatively difference choices: either i) it is optimal to choose x_i as a zero of (17), essentially ignoring the existence of firm $n(X)+1$, or ii) it is optimal to choose $x_i =$

[13] Recall that $n(X)$ is piece-wise constant, which is why there are no $n'(X)$ terms in (17).

$X^d_{n(X)+1}-X_{-i}$ at a discontinuity in marginal profit, where the firm deters the activity of firm $n(X)+1$. For the former, setting (17) equal to zero yields

$$r_{i,n(X)}(X_{-i})=\frac{n(X)-1}{2n(X)}\left(a+n(X)\bar{c}_n(X)-(n(X)+1)c_i-X_{-i}\right), \tag{18}$$

where the notation $r_{i,k}(X_{-i})$ for $k>i$, indicates the best-response for firm i when there are k active firms[14] and the deterrence of firm $k+1$ is not optimal, i.e. $r_{i,k}(X_{-i})+X_{-i}\geq X^d_{k+1}$. Consequently, a firm's best-response will either be of the form $r_{i,k}(X_{-i})$ or $X^d_k-X_{-i}$ depending on X_{-i}. The following Lemma establishes the precise nature of the best-response functions.

Lemma 2. *An active firm i's best response to X_{-i}, $r_i(X_{-i})$, is a decreasing and continuous function with the following properties:*

1. *$r_i(X_{-i})$ alternates between $r_{i,k}(X_{-i})$ and $X^d_k-X_{-i}$ for a decreasing sequence of $k>i$ as X_{-i} increases from 0 to X^d_i,*
2. *$r_i(X_{-i})=0$ for $X_{-i}\geq X^d_i$.*

Proof: See Appendix A.

We illustrate the best-response for firm $i<k$ in Figure 1. $r_{i,k}(X_{-i})$ is i's best-response when it results in $x_i<X^d_k-X_{-i}$ and $r_{i,k-1}(X_{-i})$ is i's best-response when it results in $x_i>X^d_k-X_{-i}$. For values of X_{-i} that result in $r_{i,k-1}(X_{-i})\leq X^d_k-X_{-i}\leq r_{i,k}(X_{-i})$, the best-response is $X^d_k-X_{-i}$. The overall best-response, $r_i(X_{-i})$, is the solid line connecting these three segments. A similar pattern occurs for firm i's best-response function as it crosses each threshold, $X^d_k-X_{-i}$ for each $k=i+1,...,N$.

It is interesting to contrast this best-response function with those found in the literature on entry deterrence with multiple incumbents,[15] which exhibit a similar effect when a firm finds it optimal to set a limit output. For example, Gilbert and Vives (1986) show that best-response functions are discontinuous due to the presence of fixed, entry costs that must be incurred by potential entrants. This leads to multiple equilibria with both a non-deterrence equilibrium and a range deterrence equilibria coexisting. Since we do not have fixed costs, best-responses in our model are continuous, so we get a unique *type* of equilibrium, either involving non-deterrence or deterrence, but not both simultaneously.

An immediate consequence of Lemma 2 is that a pure-strategy equilibrium exists since the best-response functions are continuous and bounded. However, it is not necessarily unique. To see this note that, if there is a deterrence equilibrium with, say, k active firms deterring firm $k+1$, each firm must be playing the best-response that deters firm $k+1$, i.e., $x_i(X_{-i})=X^d_{k+1}-X_{-i}$. So firms playing mutual best-responses results in the condition $\sum_{i=1}^k x^*_i=X^d_{k+1}$ from which we cannot determine a unique $(x^*_1,...,x^*_k)$. Of course aggregate sales are determined in such an equilibrium (at X^d_{k+1}), as is price (at c_{k+1}). The best-response functions do impose some limits on the individual advance sales since it is required that for each $i=1,...,k$, X^*_{-i} is in the range of values for which the firm wishes to choose the deterrence level of sales, but this yields a continuum of possible equilibrium sales vectors, each resulting in the same aggregate advance sales and price.

[14] Since the discontinuities in marginal profit coincide with those of $n(X)$, the number of firms is not affected by local variations in x_i in this case.

[15] Gilbert and Vives (1986), Vives (1988)

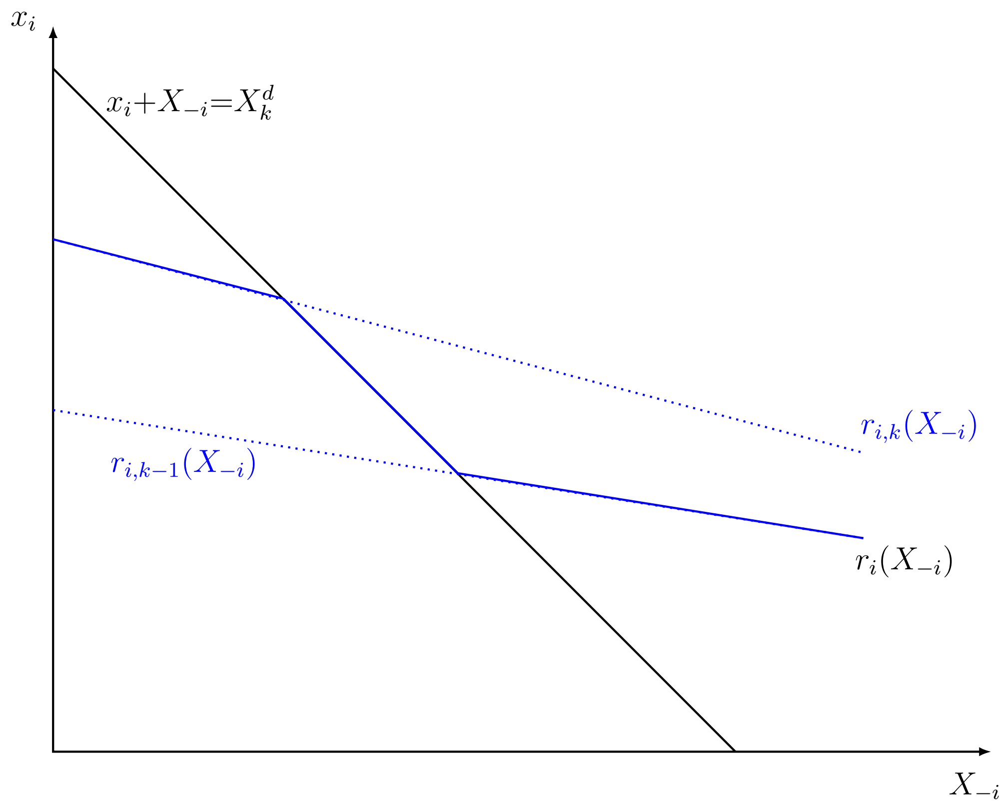

Figure 1: Best response $r_i(X\_i)$ with $k{>}i$. $r_i^k(X\_i)$ is the best-response when there are k firms active, and $x_{i,k}^d(X_{-i})$ is the threshold value for x_i that causes firm k to be inactive.

We can illustrate the nature of the alternative types of equilibria graphically by focusing on the case with three potential firms and examine the types of situations that give rise to the duopoly outcome as an equilibrium, i.e. $x_3^*{=}0$. Figure 2 plots the best-response functions for firms one and for zero sales by firm three, i.e., $r_1(x_2{+}0)$ and $r_2(x_1{+}0)$. The two panels in the figure illustrate the two possible situations in which a duopoly emerges in equilibrium. Panel (a) depicts the situation in which the equilibrium, E, has the mutual best-response of firms one and two above $x_1{+}x_2{=}X_3^d$. In this case, a unique equilibrium occurs at point E with $x_1^*{+}x_2^*{>}X_3^d$. Panel (b) of Figure 2, illustrates the other possibility. The mutual best-responses occur on the line segment EE' where there are multiple equilibria, each with $x_1^*{+}x_2^*{=}X_3^d$. The point labelled $\bar{E}$ represents the equilibrium that would occur if firm three did not exist, with lower aggregate output than in any of the equilibria along EE'.

The proof of the following proposition formally establishes these results and provides the conditions on the model parameters under which each type of equilibrium obtains.

Proposition 2. *Given a and $\mathbf{c}$, a pure-strategy, sub-game perfect Nash equilibrium exists in which the equilibrium aggregate advance sales is unique. Furthermore, there is a sequence of thresholds, $\alpha_1{<}\alpha_1^d{<}\alpha_2{<}\alpha_2^d{<}...{<}\alpha_{N-1}{<}\alpha_{N-1}^d{<}\alpha_N{<}\alpha_N^d{\equiv}\infty$, for which the equilibrium number of active firms is $n^*{=}\max\{k|a{>}\alpha_k\}$ and the equilibrium either*

i) a non-deterrence equilibrium for $a{\in}(\alpha_{n^*}, \alpha_{n^*}^d]$ with,

$$X^*{=}\frac{n^*(n^*{-}1)}{n^{*2}{+}1}\,(a{-}\bar{c}_{n^*}) \quad \textit{and} \quad p_A^*{=}\frac{a{+}n^{*2}\bar{c}_{n^*}}{n^{*2}{+}1},$$

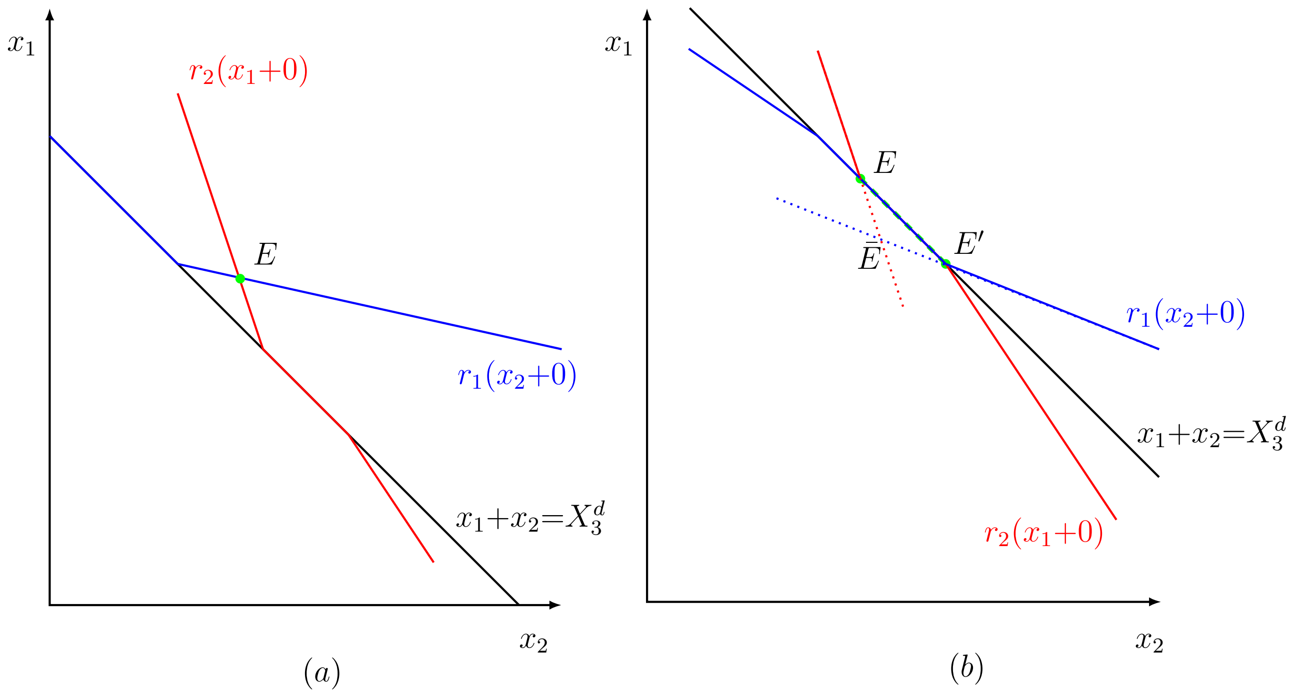

Figure 2: Best responses, three potential firms with $x_3{=}0$. (a) non-deterrence equilibrium, (b) deterrence equilibria.

or

ii) a deterrence equilibrium for $a{\in}(\alpha^d_{n^*}, \alpha_{n^*+1}]$ *with,*

$$X^*{=}X^d_{n^*+1} \quad \textit{and} \quad p^*_A{=}c_{k+1}.$$

Proof: See Appendix A.

The demand thresholds[16] in Proposition 2 are of two types. First, the *activity thresholds*, $\alpha_1{<}\alpha_2{<}...{<}\alpha_N$ determine whether a firm is active, so $a{\in}(\alpha_n, \alpha_{n+1}]$ means that there are n active firms[17] in equilibrium. The *deterrence thresholds* further subdivide these intervals, determining the nature of the equilibrium: there are n firms active who do not deter firm $n{+}1$ if $a{\in}(\alpha_n, \alpha^d_n]$ (firm $n{+}1$ is blockaded), and n firms active who deter the activity of firm $n{+}1$ if $a{\in}(\alpha^d_n, \alpha_{n+1}]$.

The activity and deterrence thresholds in Proposition 2 are derived in the proof from conditions on aggregate advance sales. However, we can provide an alternative, intuitive argument based on the equilibrium price. A firm will be active in equilibrium if the price exceeds its marginal cost. Given our ranking of marginal costs, it is clear that if firm $j{>}i$ is active, then so is firm i. Let $p^{nd}_A(n){=}(a{+}n^2\bar{c}_n)/(n^2{+}1)$ denote the expression for price in a non-deterrence equilibrium with n active firms. For all N firms to be active it must be that the equilibrium price with N firms active, $p^{nd}_A(N)$, exceeds the marginal cost of

[16] We present these as thresholds for the demand parameter a, but the α_k and α^d_k are themselves determined by the vector of marginal cost parameters, so these conditions represent subsets of the feasible parameters.

[17] We drop the * from the equilibrium number of firms when it is unambiguous in order to reduce notational clutter.

all firms, in particular that of the least efficient firm, N. Firm N's activity threshold follows directly from $p_A^{nd}(N){>}c_N$:

$$a{>}c_N{+}N^2(c_N{-}\bar{c}_N){\equiv}\alpha_N. \tag{19}$$

Similarly, for an equilibrium to have $n{<}N$ active firms and $N{-}n$ inactive firms requires $p_A^{nd}(n){>}c_n$ and $p_A^{nd}(n{+}1){\leq}c_{n+1}$, i.e. the marginally active firm earns a positive margin, while the marginally inactive firm would not face a positive margin were it to be active. Again, we can express these two conditions in terms of the activity thresholds of firms n and $n{+}1$:

$$\alpha_n{\equiv}c_n{+}n^2(c_n{-}\bar{c}_n){<}\, a\, {\leq}c_{n+1}{+}(n{+}1)^2(c_{n+1}{-}\bar{c}_{n+1}){\equiv}\alpha_{n+1}. \tag{20}$$

In deriving the α_n's in this way, we implicitly assume that the equilibrium is of the non-deterrence type when a is slightly larger than α_n since we use $p_A^{nd}(n)$ at this point. To see this that this is valid, note that the limit of $p_A^{nd}(n)$ as a approaches α_n is c_n, which is strictly less than c_{n+1}, so the equilibrium is not a deterrence one. For $a{>}\alpha_n$ as long as $p_A^{nd}(n){<}c_{n+1}$ the equilibrium is as described in part i) of Proposition 2. However, if a is such that $p_A^{nd}(n){=}c_{n+1}$, firm $n{+}1$ is not indifferent between being active versus inactive, since $p_A^{nd}(n{+}1){<}p_A^{nd}(n)$ as the activity of firm $n{+}1$ causes a discrete reduction in price. So for values of a slightly larger than that which results in $p_A^{nd}(n){=}c_{n+1}$, price must remain at c_{n+1} as otherwise firm $n{+}1$ becomes active. The deterrence threshold, α_n^d follows immediately from $p_A^{nd}(n){=}c_{n+1}$:

$$a{=}c_{n+1}{+}n^2(c_{n+1}{-}\bar{c}_n){\equiv}\alpha_n^d. \tag{21}$$

Hence, the activity threshold for firm k can be derived as the value of a for which $p_A^{nd}(k){=}$ c_k and the deterrence threshold where k firms deter the activity of firm $k{+}1$ as the value of a for which $p_A^{nd}(k){=}c_{k+1}$.

Proposition 2 establishes that the advance price and aggregate advance sales are unique in the sub-game perfect equilibrium. In the next section, we compare the equilibrium price to the benchmarks of the homogeneous firm game with advance sales as well as the static Cournot game in order to determine the implications for the competitiveness of the equilibrium.

4 Competitiveness: the effect on price and the number of active firms

To see how the equilibrium price and number of active firms vary with model parameters, we illustrate how price varies with demand in Figure 3 for $N{\geq}4$. Equilibrium price increases with a in a step-like pattern, which is due to the change in the type of equilibrium as demand varies (for fixed marginal costs). The equilibrium alternates between non-deterrence and deterrence as successively higher-cost firms are alternately blockaded, deterred, and then accommodated as a increases. The dashed lines $N{=}2$ and $N{=}3$ illustrate the counterfactual price that would occur with only two or three potential firms. The effect of deterrence on price is shown by the distance from the dotted line to the flat part of the equilibrium price. For instance, for $a{\in}(\alpha_2^d, \alpha_3)$ the distance from the $N{=}2$ dashed line to c_3 represents the reduction in price caused by the deterrence behaviour of firms one and two.

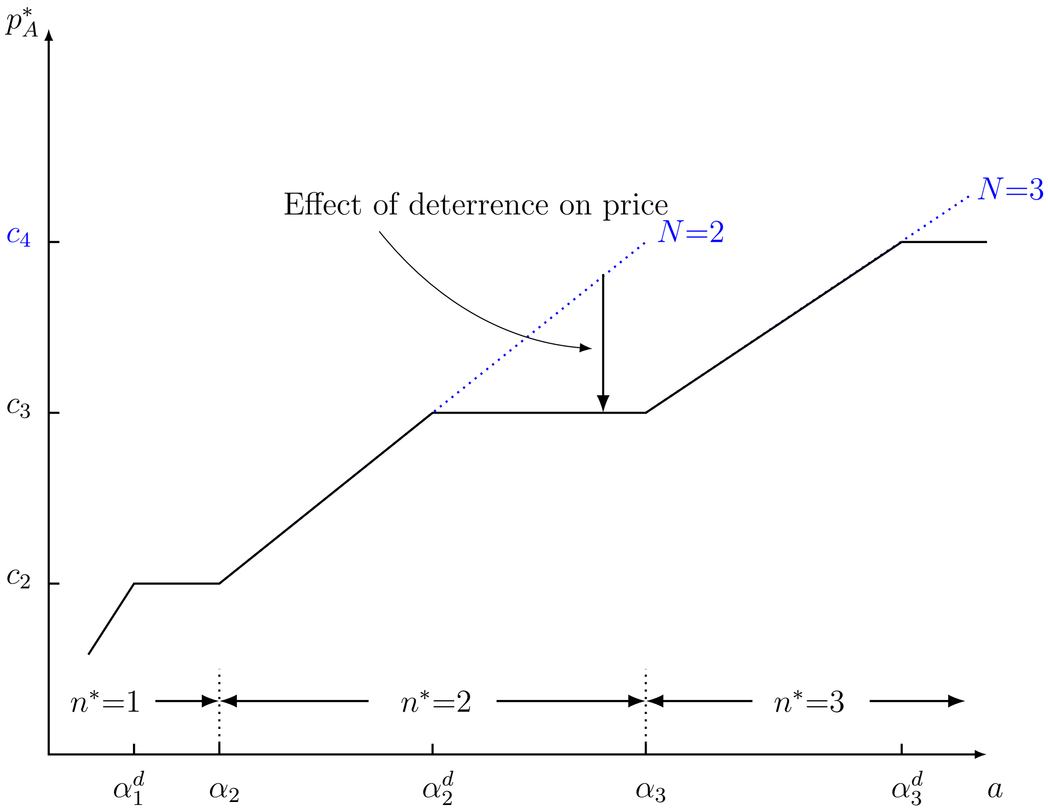

Figure 3: Equilibrium price versus a.

We can quantify the effect that deterrence has on price by simply examining $p_A^{nd}(n) - c_{n+1}$ on the range $\alpha_n^d < a \leq \alpha_{n+1}$. This difference is zero at the lower end of this interval as $p_A^{nd}(n) = c_{n+1}$ is what defines α_n^d. At the upper end of the interval, the price difference is the largest:

$$\frac{\alpha_{n+1} + n^2 \bar{c}_n}{n^2+1} - c_{n+1} = \frac{n}{n^2+1}(c_{n+1} - \bar{c}_n). \tag{22}$$

Conditional on remaining in the deterrence region, the higher the marginal cost of the deterred firm, the larger the maximum price reduction due to deterrence. Furthermore, for a given cost disadvantage of the deterred firm, $c_{n+1} - \bar{c}_n$, this price difference is decreasing in n, so the effect of deterrence is more significant the fewer active firms there are. We present some numeric computations of the size of this maximal price difference in Table 2 of Appendix B.

An interesting result occurs when $a \in (\alpha_1^d, \alpha_2]$. The equilibrium in this case has firm one deterring firm two (and blockading all higher cost firms) by choosing sales that induces $p_A^* = c_2$. This is precisely the outcome in the heterogeneous cost Bertrand game, in which the firm with the lowest cost sets its price at the marginal cost of the next-lowest cost rival. In this case, the equilibrium mimics the Bertrand equilibrium with only one period of advance sales, in contrast with the result in Allaz and Vila (1993) in which marginal cost pricing obtains only in the limit as the number of forward trading periods increases. In their homogeneous firm model, marginal cost pricing is the efficient outcome, whereas in our model full efficiency is not obtained since $p_A = c_2 > c_1$ is the most efficient outcome that can be obtained.

4.1 Comparison to the homogeneous firm model

To answer the question of how heterogeneity affects the equilibrium relative to the homogeneous firm case, a benchmark homogeneous firm model is required for the comparison. A natural candidate for a benchmark is to have homogeneous firms with marginal cost equal to the average of the marginal costs in the heterogeneous firm model. However, there are two possible interpretations of this: the average marginal cost of active firms, $\bar{c}_{n^*}$, or the average marginal cost of potential firms, $\bar{c}_N$. We examine both in turn, beginning with the homogeneous n-firm benchmark.

The equilibrium price in an equilibrium with n homogeneous firms, each with marginal cost, c, is $\bar{p}_A(n,c)=\frac{a+n^2c}{n^2+1}$,[18] which is of the same form as the non-deterrence equilibrium price in Proposition 2, but with the common marginal cost c in place of $\bar{c}_n$: $\bar{p}_A(n,\bar{c}_n)=p_A^{nd}(n)$. Consequently, if the equilibrium is a non-deterrence one with n firms, i.e., $a\in[\alpha_n,\alpha_n^d)$, then firm heterogeneity does not affect the competitiveness of the equilibrium compared to the homogeneous firm model with the same number of firms and average marginal cost. However, in a deterrence equilibrium with n active firms, i.e., $a\in[\alpha_n^d,\alpha_{n+1})$, price will be lower than in the homogeneous firm benchmark since deterrence results in $p_A^*=c_{n+1}<p_A^{nd}(n)=\bar{p}_A(n,\bar{c}_n)$. This establishes the following:

Proposition 3. *The advance sales equilibrium with heterogeneous firm is no less competitive than the homogeneous firm benchmark with the same number of firms, and is strictly more competitive if the equilibrium is a deterrence one.*

Consider now the benchmark homogeneous N-firm equilibrium with marginal cost $c=\bar{c}_N$. The benchmark price is $\bar{p}_A(N,\bar{c}_N)=\frac{a+N^2\bar{c}_N}{N^2+1}$ as long as $a>\bar{c}_N$, however, this is the equilibrium price with heterogeneous firms only if $a>\alpha_N>\bar{c}_N$. Consequently, for $a\in(\bar{c}_N,\alpha_N)$, there are fewer firms active in the heterogeneous firm case, $n<N$. If the equilibrium is a non-deterrence one the equilibrium price is $p_A^*(n)=\frac{a+n^2\bar{c}_n}{n^2+1}$, which can be higher or lower than $\bar{p}_A(N,\bar{c}_N)$ depending on two offsetting factors. Fewer firms active ($n<N$) leads to less competitive behaviour, and hence, a higher price, however the average marginal cost of active firms is lower ($\bar{c}_n<\bar{c}_N$), leading to a lower price, so we cannot sign the effect on price in general. We illustrate below in an example that either effect can dominate, however, we can determine the effect in the specific case in which $N-1$ active firms deter firm N:

Proposition 4. *If firm N is deterred in equilibrium, i.e., $a\in[\alpha_{N-1}^d,\alpha_N)$, the equilibrium price exceeds that of the homogeneous firm advance sales game in which $c=\bar{c}_N$.*

Proof. Since $\bar{p}_A(N,\bar{c}_N)=\frac{a+N^2\bar{c}_N}{N^2+1}$, at $a=\alpha_N$, $\tilde{p}_A(N,\bar{c}_N)=p_A^*=c_N$ by the definition of α_N. For $a\in[\alpha_{N-1}^d,\alpha_N)$, p_A^* remains equal to c_N, but $\tilde{p}_A(N)<c_N$ since it is increasing in a and $a<\alpha_N$ on $[\alpha_{N-1}^d,\alpha_N)$. $\square$

Proposition 4 establishes that the advance sales equilibrium with heterogeneous firms is "less competitive" than the homogeneous N-firm benchmark when conditions result in the deterrence of firm N. We can also establish that there exist demand and cost conditions in which the opposite occurs. The simplest case in when firm one deters firm two, i.e., $a\in[\alpha_1^d,\alpha_2)$, which as mentioned above, mimics the Bertrand equilibrium with

[18] See Bushnell (2007) and Miller and Podwol (2020).

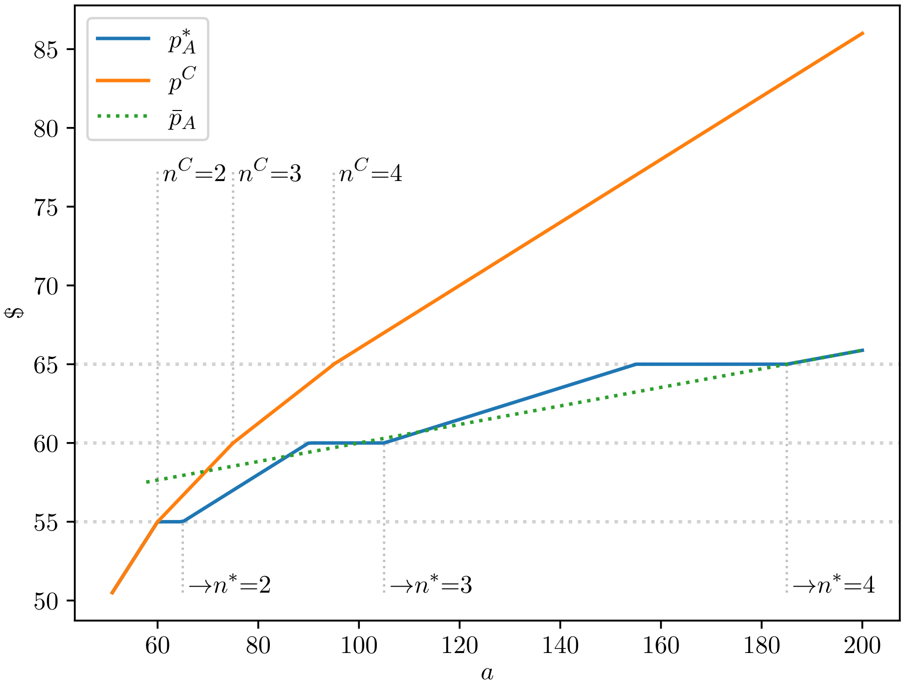

Figure 4: Equilibrium price, p_A^*, Cournot price, p^C, and homogeneous firm price, $\bar{p}^A$ versus a. Heterogeneous costs: $\boldsymbol{c}$=(50, 55, 60, 65); homogeneous cost c=57.5.

p_A^*=c_2, clearly lower than the price in the homogeneous firm case as long as $c_2<\bar{c}_N$, which is clearly feasible under our assumptions. Of course, this also requires $a>\bar{c}_N$, i.e., demand is sufficient for the homogeneous firm equilibrium to be feasible. Consequently, depending on demand and cost conditions, equilibrium price with heterogeneous firms may be higher or lower than that with homogeneous firms. We next present an illustrative example to demonstrate that this is possible.

4.2 Illustration

We now illustrate the above results with an example in which we consider four potential producers with marginal costs given by $\boldsymbol{c}$=(50, 55, 60, 65), and consider a varying from 55 to 200. These numbers are chosen simply because they generate figures in which the alternative equilibria are clearly discernible. The equilibrium price, p_A^*, is plotted in Figure 4, along with the corresponding Cournot price, p^C, and the price in the homogeneous firm advance sales equilibrium where each of the four firms has a marginal cost of 57.5: $\bar{p}_A(4, 57.5)$.

Comparing the advance sales price with that from the homogeneous firm game, we see that the results of Proposition 4 and the discussion following it bear out. The equilibrium is less competitive with heterogeneous firms for demand levels in which the three most efficient firms deter firm four, $a\in[155, 185]$, and more competitive at low levels of demand in which firm one deters firm two, $a\in[60, 65]$. Moreover, the relative competitiveness of the two models varies for levels of demand intermediate between these two.

Figure 4 also demonstrates the dramatic difference in the number of active firms between the advance sales model and the heterogeneous firm Cournot model. All firms are active in the Cournot model at significantly lower levels of demand than in the advance sales model, $n^C{=}4$ for $a{>}95$ or so, and $n^*{=}4$ for $a{>}185$. In addition, there is no level of demand for which three firms are active in both models, so any comparative analysis must consider this variation in the active number of firms when considering the effects of advance sales.

5 Efficiency: the effect on industry average cost

The distribution of sales amongst active firms determines the efficiency of aggregate production since firms differ in their marginal cost, but clear results on the effects are complicated by the multiple equilibria when the equilibrium is a deterrence one. However, we can determine bounds on individual production in the deterrence case. We next establish the details of individual firm advance sales and then discuss the implications for average production cost.

5.1 Equilibrium individual advance sales

Proposition 2 established the unique equilibrium *aggregate* advance sales, but in the proof we note that individual advances sales are only uniquely determined in a non-deterrence equilibrium. In the deterrence case, there is a continuum of equilibrium individual advance sales allocations,[19] however we can establish bounds on each firm's level of sales in such an equilibrium. Figure 2(b) provides an illustration of these bounds as the point E is at the maximal sales for firm one and the minimal sales for firm two and the point E' is at the minimal sales for firm one and the maximal sales for firm two. The following proposition states the results:

Proposition 5. *In an advance sales equilibrium with n active firms, individual advance sales are determined as follows:*

a) If $a{\in}(\alpha_n, \alpha_n^d]$, individual sales of active firms are unique and given by

$$x_i^*{=}(n{-}1)\left(\frac{a{+}n^2\bar{c}_n}{n^2{+}1}{-}c_i\right), \quad \forall i{\leq}n.$$

b) If $a{\in}(\alpha_n^d, \alpha_{n+1}]$, individual sales of active firms must satisfy the following conditions:

$$\sum_{i=1}^{n} x_i^*{=}X_n^d,$$

and

$$x_i^*{\in}[(n{-}1)(c_{n+1}{-}c_i), n(c_{n+1}{-}c_i)], \qquad \forall i{\leq}n.$$

[19]This also occurs with entry deterrence by multiple incumbents as in Gilbert and Vives (1986) and Vives (1988).

Proof: See Appendix A.

Proposition 5 shows that the range of possible advance sales for a firm in a deterrence equilibrium is increasing in the efficiency of that firm. Indeed, the size of the interval of potential advance sales for firm i is $c_{n+1}-c_i$, so more efficient firms have a larger range of possible sales. In addition, both the maximum sales are higher for more efficient firms and the minimum sales are higher for more efficient firms.

5.2 Non-deterrence equilibrium: Total sales and market shares

We now analyze total production by each firm in order to determine how advance sales affects relative market shares. Since individual firms' advance sales are only uniquely defined in a non-deterrence equilibrium, we first focus on this case.

In a non-deterrence equilibrium with n active firms, i.e., $a \in (\alpha_n, \alpha_n^d]$, we can determine total sales for active firms:

$$x_i^* + y_i^*(X^*) = n\left(\frac{a-\bar{c}_n}{n^2+1} + \bar{c}_n - c_i\right), \tag{23}$$

Furthermore, aggregate spot sales are

$$Y^* = \sum_{i=1}^{n} y_i^*(X^*) = \frac{n(a-\bar{c}_n)}{n^2+1}, \tag{24}$$

so the market share of an active firm is then

$$s_i \equiv \frac{x_i^* + y_i^*(X^*)}{X^* + Y^*} = \frac{1}{n} + \frac{(\bar{c}_n - c_i)(n^2+1)}{n(a-\bar{c}_n)}. \tag{25}$$

A firm whose marginal cost is equal to the average of the marginal costs of active firms has a share of total sales equal to $1/n$. Firms that are more efficient than the average of active firms, $c_i < \bar{c}_n$, have market shares larger than $1/n$ while those that are less efficient than average have market shares lower than $1/n$. We established above that the equilibrium price in such a non-deterrence equilibrium is the same as that in a homogeneous firm game in which each firm has a marginal cost $c = \bar{c}_n$. However, in the homogeneous firm game, each firm has a market share of $1/n$, so compared to that equilibrium, sales are increased for relatively efficient firms and decreased for relatively inefficient firms. Consequently, industry average cost is lower and there is a welfare gain in the heterogeneous firm model compared to the homogeneous firm model. Summarizing, we have

Proposition 6. *In a non-deterrence equilibrium with n active firms, the average cost of producing total output is lower than in the homogeneous firm advance sales equilibrium with $c = \bar{c}_n$.*

The pattern in firms' market shares is similar to that which occurs in the heterogeneous firm Cournot equilibrium, so it is interesting to compare the two. Since the number of firms potentially differ in the Cournot and advance sales equilibria, we focus here on the case in which all N potential firms are active,[20] so there is no possibility of a limit equilibrium in the advance sales game.

[20] From (19), for firm N to be active requires $a > c_N + N^2(c_N - \bar{c}_N)$, which is also sufficient for N firms to be active in the Cournot equilibrium, which requires $a > c_N + N(c_N - \bar{c}_N)$.

Using Corollary 1, market shares in the Cournot game with N firms are

$$s_i^C = \frac{y_i^*(0)}{\sum_{i=1}^N y_i^*(0)} = \frac{1}{N} + \left[\frac{\bar{c}_N - c_i}{N(a - \bar{c}_N)}\right](N+1). \tag{26}$$

In the Advance Sales game, using (25) with all firms active we have

$$s_i = \frac{1}{N} + \left[\frac{\bar{c}_N - c_i}{N(a - \bar{c}_N)}\right](N^2+1). \tag{27}$$

Comparing the two market shares we find

$$s_i - s_i^C = \frac{\bar{c}_N - c_i}{(a - \bar{c}_N)}(N-1), \tag{28}$$

from which we see that relatively efficient firms (those with $c_i < \bar{c}_N$) have higher market shares in the advance sales equilibrium, whereas relatively inefficient firms (those with $c_i > \bar{c}_N$) have lower markets shares in the advance sales equilibrium. Notice that we get the same result for any situation in which $n_C = n^* < N$, however, there may not be a feasible set of parameters that generates this outcome. Hence, advance sales result in a reallocation of market share towards relatively efficient firms compared to the Cournot equilibrium, from which we have the following:

Proposition 7. *In a non-deterrence equilibrium of the advance sales game, if $n^* = n_C$, the average cost of production is lower in than in the Cournot equilibrium.*

Clearly, since the market shares of the largest (most efficient) firms increase and that of the smallest (least efficient) firms decrease, advance sales have the effect of increasing measured concentration in the market, which we state as

Corollary 2. *If $n^* = n_C$, advance sales result in an increase in the HHI relative to the Cournot equilibrium.*

Even though measured concentration is higher than in the Cournot equilibrium when advance sales are possible, both price and average cost are lower, resulting in an unambiguous welfare gain.

5.3 Deterrence equilibria: bounds on individual firm sales

The range of possible individual firm outputs in a deterrence equilibrium suggests that there are a variety of levels of efficiency of the possible equilibria. Clearly, ones where the most efficient firms produce as much as possible are the most efficient. To examine the potential degree of inefficiency consider two firms $i < j$ active in a deterrence equilibrium. From Proposition 5b), the minimum sales by the more efficient firm, i, is $(n-1)(c_{n+1} - c_i)$, and the maximum sales by the less efficient firm, j, is $n(c_{n+1} - c_j)$, so it is possible that j sells more than i if $c_j - c_i < (c_{n+1} - c_i)/n$. Since there is no restriction on the size of $c_j - c_i$ beyond it being positive, and $c_{n+1} - c_i > 0$, we have established the following Corollary to Proposition 5:

Corollary 3. *In a deterrence equilibrium, it is possible that a less-efficient firm sells as much or more in advance than a more efficient firm, i.e. $x_i^* \leq x_j^*$, for $i < j \leq n$.*

Since the aggregate advance sales and price in a deterrence equilibrium are the same for any of the possible combinations of individual advance sales, we cannot use the Pareto criterion to identify a focal equilibrium. However, we can use the restrictions on individual firm sales in Proposition 5b to define two particular deterrence equilibria of interest: the most- and least-efficient equilibria. These represent the best-case and worst-case scenarios for a deterrence equilibrium with n firms since they correspond to the lowest- and highest-cost allocations for producing the aggregate quantity X_n^d, and can be defined as follows:[21]

Definition 2. *The* **most-efficient deterrence equilibrium** *is that in which advance sales are concentrated in efficient firms as much a possible while still satisfying the conditions in Proposition 5b, whereas the* **least-efficient deterrence equilibrium** *is that in which advance sales are concentrated in inefficient firms as much as possible.*

In the case of firms one and two deterring firm three, the most- and least-efficient deterrence equilibria are illustrated in panel (b) of Figure 2 by points E and E' respectively.

With the most- and least-efficient deterrence equilibria defined, we can extend the computed example of subsection 4.2 to illustrate how individual sales vary with the level of demand. We plot individual advance sales in Figure 5 with only the most- and least-efficient equilibria plotted when the equilibrium involves deterrence. Notice that, in the least-efficient case, there are demand levels where $x_1^*{=}x_2^*$, i.e., a less efficient firm sells as much in advance as does a more efficient firm, consistent with Corollary 3.[22] Also notice that the magnitude of the difference between the maximal and minimal sales for a firm in a deterrence equilibrium is quite significant. For example, in the equilibrium in which firms one and two deter the activity of firm 3 (for values of a roughly between 90 and 105), there are demand levels in which firm two sells roughly 5 units in the efficient equilibrium while selling double that in the least-efficient equilibrium.

Proposition 7 established that the industry average cost of production was lower in a non-deterrence equilibrium than in the Cournot equilibrium, conditional on their being with the same number of active firms in each. We can now present a full comparison of industry average cost between the two models, using the most- and least-efficient deterrence equilibrium to bound the outcome in the deterrence case. The industry average cost of production in equilibrium is

$$AC^*{=}\frac{\sum_{i=1}^n c_i(y_i^*(X^*){+}x_i^*)}{X^*{+}Y^*}. \tag{29}$$

Applying this to our illustration, we plot average cost in Figure 6 as the level of demand varies. This figure also plots the average cost of sales in a Cournot equilibrium for the same level of demand and so, incorporates the potentially different number of active firms between the two models. Even in the least-efficient deterrence equilibria, average cost in this example is well below that in the Cournot model. The magnitude of the different is not insignificant in proportional terms: the maximal percentage difference between the Cournot case and the efficient equilibrium is 6.2% in Figure 6. Not surprisingly, given the differences across alternative deterrence equilibria that we see in Figure 5, there is also a fairly large range of potential average costs when the equilibrium involves deterrence.

[21] A more formal definition s provided in Appendix C.

[22] It can be shown that if the difference between successive firms marginal costs is constant, as in this example, the result in Corollary 3 holds only with equality. A less efficient firm selling strictly more than a more efficient firm can happen for other, less uniform, distributions of marginal cost among firms.

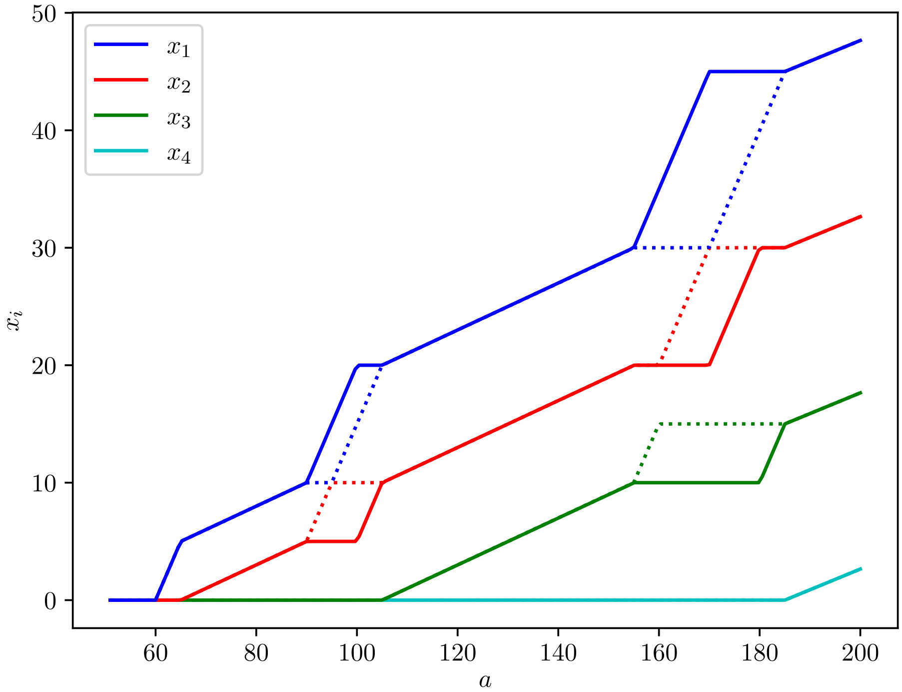

Figure 5: Advance sales by firm versus a for $\boldsymbol{c}$=(50, 55, 60, 65). Most-efficient deterrence equilibrium (solid lines) and least-efficient deterrence equilibium (dotted lines).

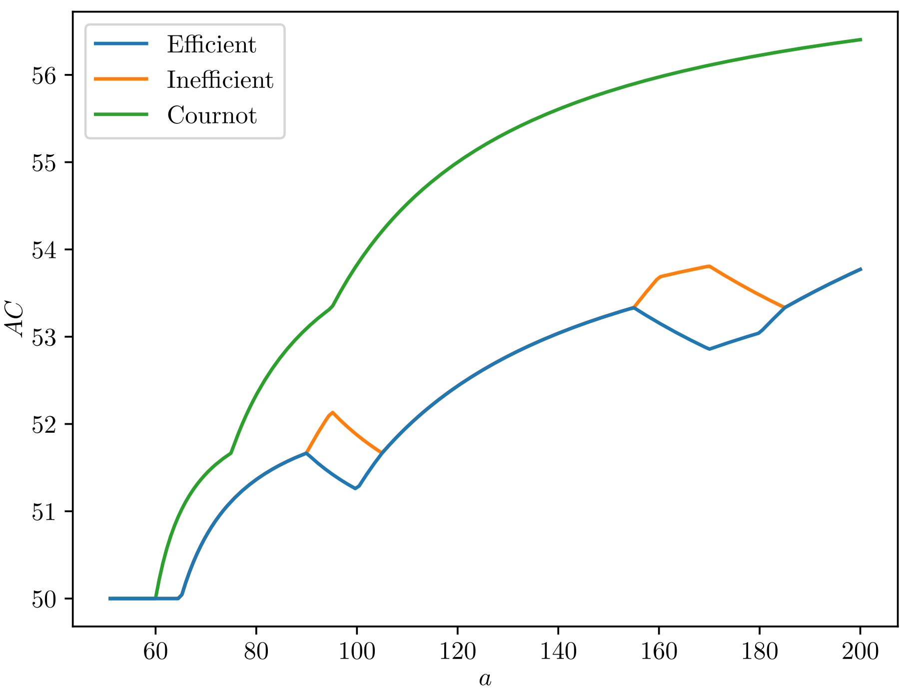

Figure 6: Industry average cost versus a for $\boldsymbol{c}$=(50, 55, 60, 65).

6 Implications for competition policy

The effects of advance sales on both the number of active firms and the competitiveness of the equilibrium relative to the Cournot equilibrium has implications for the relationship between concentration and market power, and also for merger analysis. We discuss each of these in turn.

6.1 Concentration and market power

Compared to the Cournot equilibrium, we have shown that there are (weakly) fewer firms active in the equilibrium with advance sales, and also, from (28) that more efficient, and hence, larger, firms have larger market shares than their Cournot counterparts when the number of active firms is the same in the two models. Both of these imply that measured concentration will be higher for the advance sales equilibrium than for the Cournot equilibrium. Although this reallocation of market share to more efficient firms means that firms with higher price-cost margins increase their share of sales, margins themselves are reduced due to the advance sales equilibrium being more competitive than the Cournot equilibrium. We can examine the net effect by computing the Lerner Index for the two models: $L=\sum_{i=1}^{n^*} s_i(p^*-c_i)/p^*$. To see the net effect for our illustration, we plot the HHI and the Lerner Index in Figures 7 and 8. Clearly the effect of lower price-cost margins dominates in this case as the Lerner Index is substantially below that in the Cournot equilibrium, even though the HHI is higher.

The Lerner Index is also substantially higher than that in the N-firm homogeneous firm advance sales game due to the fact that more efficient firms have both higher market shares and higher price-cost margins than in the homogeneous firm case. However, this higher Lerner Index does not necessarily correspond to a welfare loss as price levels can be higher, lower, or the same in the two models, as we see from Figure 4. In addition, the reallocation of sales in the heterogeneous firm case results in lower industry average cost that offsets a loss in consumer surplus in the event of an increase in price relative to the homogeneous firm game.

6.2 Mergers

Although our model is not designed for the analysis of mergers,[23] the possibility of a deterrence equilibrium being the outcome either pre- or post-merger raises some novel implications of mergers among firms with constant and heterogeneous marginal costs. Miller and Podwol (2020) are the only ones to our knowledge that examine the effects of mergers in a model of strategic forward trading. They use a cost function similar to that pioneered in Perry and Porter (1985), where firms differ only in the slope of their linear marginal cost function. In their model, a merger between firms results in a lowered marginal cost for the merged firm when it produces positive output. However, all firms have the same minimum marginal cost: $C_i'(0)=c$. Consequently, there is no deterrence

[23] Salant et al. (1983) demonstrated that mergers among identical firms with constant marginal cost and no capacity constraints are not profitable for the merging parties. Beginning with Perry and Porter (1985), a number of papers criticized the use of this cost structure as it implies that the merged firm is identical to any of the pre-merger firms.

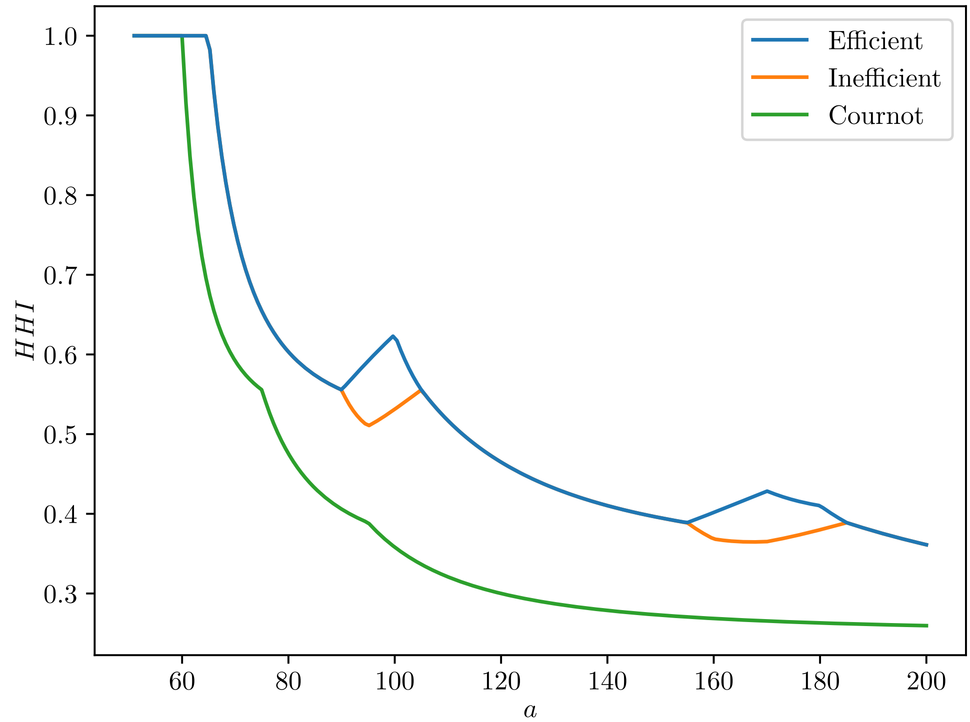

Figure 7: Hirschmann-Herfindahl Index versus a for $\boldsymbol{c}$=$(50, 55, 60, 65)$.

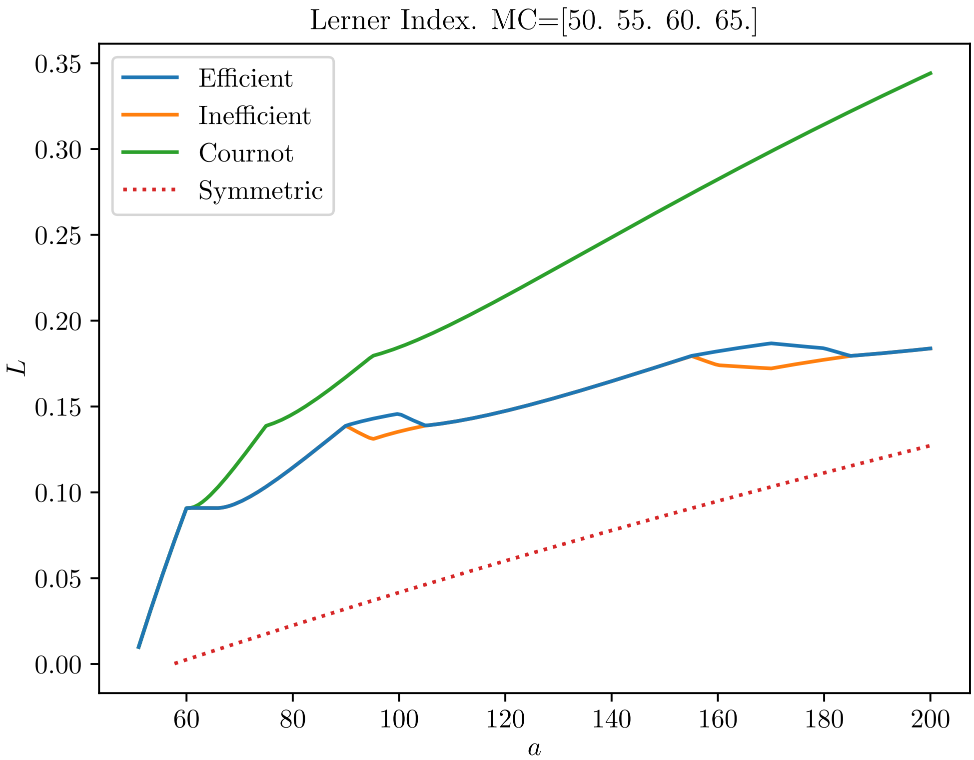

Figure 8: Lerner Index versus a for $\boldsymbol{c}$=$(50, 55, 60, 65)$.

possible and all firms are active, so pre- and post-merger equilibria are analogous with the non-deterrence equilibria in our model. With our constant, but heterogeneous, marginal cost, the possibility of deterrence equilibria complicates the analysis of a merger as both the pre- and post-merger equilibria can be either a deterrence or a non-deterrence one. A complete analysis all the possible permutations is beyond the scope of this paper, however, one particular type of merger is worth discussing: that between an active firm and an inactive firm.

Clearly mergers between active and inactive firms will have no effect if the pre-merger equilibrium is a non-deterrence one or if the merger involves an inactive firm with higher costs than the firm being deterred. However, if the pre-merger equilibrium involves the deterrence of firm $n{+}1$, an active firm acquiring firm $n{+}1$ will induce an increase in price (assuming that the new firm operates with the more efficient firm's marginal cost), even though post-merger, the number of active firms remains the same. The merger leads to a price higher than the pre-merger price of c_{n+1} (either the non-deterrence price with n firms, or the marginal cost of the next highest cost firm, c_{n+2}), and so to reduced consumer surplus. However, the calculation of the full welfare effects are clouded by the ambiguity in computing the effect on average cost due to the multiplicity of equilibria in the pre-merger equilibrium. Average production costs can either increase or decrease depending on whether the pre-merger equilibrium was a relatively inefficient or a relatively efficient deterrence equilibrium. If the pre-merger equilibrium is the most-efficient deterrence equilibrium, then a merger with firm $n{+}1$ causes a reallocation of market share away from relatively efficient firms along with the reduction in aggregate sales, so a merger is certainly welfare-reducing. If the pre-merger equilibrium is the least-efficient deterrence equilibrium, then the reallocation of sales to relatively efficient firms might offset the effects of the lower aggregate sales and the sign of the net effect depends on the particular circumstance.

Consider the profitability of an active firm merging with firm $n{+}1$ from a deterrence equilibrium.[24] Since firm $n{+}1$ earns zero profit pre-merger, such a merger is profitable for the merging parties if the active firm sees an increase in profit by the elimination of firm $n{+}1$. A sufficient condition for the active firm in the merger to increase its profit with the elimination of firm $n{+}1$ is that its total sales, $x_i{+}y_i^*(X)$ increase. Spot sales, $y_i^*(X)$, increase for all firms due to the reduction in aggregate advance sales caused by the merger, so an even weaker sufficient condition for profit to increase is that the firm's advance sales increase. In a deterrence equilibrium, a firm is most likely to increase advance sales following a merger if it finds itself selling the minimum quantity defined in Proposition 5, from which we establish

Proposition 8. *Any firm selling its minimum quantity, $x_i^*{=}n(c_{n+1}{-}c_i)$, in a deterrence equilibrium increases its profit by merging with firm $n{+}1$.*

Proof. Profit to firm i pre-merger when selling $x_i^*{=}n(c_{n+1}{-}c_i)$ is $\pi_i^{pre}{=}n(c_{n+1}{-}c_i)^2$. If post-merger there is a non-deterrence equilibrium with n firms, profit is $\pi_i^{post}{=}n(\frac{a+n^{*2}\bar{c}_{n^*}}{n^{*2}+1}{-}c_i)^2{=}n(p_A^{nd}(n){-}c_i)^2$. Alternatively, if post-merger there is a deterrence equilibrium with firm $n{+}2$ deterred, profit is $\pi_i^{post}{=}n(c_{n+2}{-}c_i)^2$. Both $p_A^{nd}(n){>}c_{n+1}$ and $c_{n+2}{>}c_{n+1}$, so post-merger profit is higher in either cases. □

[24] It does not matter which active firm merges with firm $n{+}1$, the effect on profits of all active firms is the same.

Proposition 8 suggests that the firms may have an incentive to break out of a deterrence equilibrium by way of merging with the deterred firm. From the discussion above, the welfare effects of this result are situation-dependent.

For a concrete example, consider the illustration described in subsection 4.2 where the equilibrium has the three most efficient firms deterring firm four, which occurs for $a \in [155, 185]$ in this example. Using the mid-point of this interval, $a=170$, the effects of a merger are presented in Table 1. For the pre-merger equilibrium, we consider both the least-efficient and the most-efficient deterrence equilibrium, labelled LEDE and MEDE respectively. From the least-efficient deterrence equilibrium, firms one and three each see an increase in profit, whereas from the most-efficient deterrence equilibrium, firms two and three see an increase in profit. So in each case, there are firms that gain and so would be willing to merge with firm four. The effect on total surplus (consumer surplus plus aggregate profit) depends on the pre-merger equilibrium. Consumer surplus declines by the same amount in each case, but the increase in total profit is higher when in the inefficient pre-merger equilibrium, resulting in an increase in total surplus in this case. If the pre-merger equilibrium is the most-efficient one, then the merger reduces the efficiency of the allocation of production across firms, resulting in a smaller increase in profit and, hence, a reduction in total surplus due to the merger.

	Pre-merger equilibrium	
	LEDE	MEDE
Price	2.3%	2.3%
Profit		
Firm 1	21.0%	-9.2%
Firm 2	-0.8%	32.3%
Firm 3	27.0%	69.3%
Total	14.0%	5.1%
Industry average cost	-0.4%	1.3%
Consumer surplus	-2.8%	-2.8%
Total surplus	0.1%	-1.4%

Table 1: Effects of merger from a three-firm deterrence equilibrium for $a=170$ and $\boldsymbol{c}=(50, 55, 60, 65)$. LEDE = least-efficient deterrence equilibrium; MEDE = most efficient deterrence equilibrium.

7 Concluding remarks

The consideration of heterogeneous marginal costs in a simple model of advance sales results an alternative type of equilibrium possible in which lower-cost active firms deter a higher-cost rival from producing. In this case inactive, higher-cost firms still exert an influence on the equilibrium. In addition, the combination of heterogeneous costs and advance sales has significant effects on market shares, skewing them towards more efficient firms. These results were derived using a simple model with linear demand and constant marginal cost. We conclude with a discussion of the potential robustness of these results to the underlying modelling assumptions.

First, the linear demand assumption is important for generating the closed-form solutions for equilibrium variables. However, the key feature driving much of the analysis is the discontinuous nature of marginal profit when price reaches a level at which a higher-cost firm becomes active. This feature is likely present under more general demand specifications due to the discrete change in the number of active firms that occurs at these points.

The assumption of constant marginal cost can also be relaxed, but the deterrence equilibria rely on firms' marginal cost at zero production be different, not that marginal cost is constant for all levels of output. For purely quadratic costs, $C_i(q_i)=c_i q_i^2$, for example, the deterrence equilibria will not exist as it is not possible to deter the activity of any firm with a non-zero price. However, as long as $C_i'(0)$ differs among firms, deterrence equilibria potentially exist.

Finally, the model analyzed here was deterministic. It is natural to question how the introduction of uncertainty affects the results, especially since advance sales, whether via contracting or storage, likely arise as a response to uncertainty. In the case of advance sales as storage by speculators, Mitraille and Thille (2020) have shown that the nature of equilibrium depends on the degree of uncertainty. If uncertainty is "minor" then certainty equivalence holds and the results are qualitatively the same as in the deterministic case. We explore whether similar results translate to the heterogeneous firm case in a companion paper.

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A Proofs

Proof of Proposition 1.

To find the equilibrium in our heterogeneous firm game, we apply the technique to solving aggregative games established in Novshek (1984), Nocke and Schutz (2018), and Anderson et al. (2020).

The first order condition for the maximization of (4) directly gives the continuous best response

$$y_i(Y_{-i},X)=\begin{cases} \frac{a-X-c_i}{2}-\frac{1}{2}Y_{-i} & \text{if } Y_{-i}<a-X-c_i \\ 0 & \text{if } Y_{-i}\geq a-X-c_i \end{cases} \tag{30}$$

Adding Y_{-i} to both sides of this expression gives the total spot sales compatible with the optimization behaviour of firm i, $Y(Y_{-i},X)$

$$Y(Y_{-i},X)=\begin{cases} \frac{a-X-c_i}{2}+\frac{1}{2}Y_{-i} & \text{if } Y_{-i}<a-X-c_i \\ Y_{-i} & \text{if } Y_{-i}\geq a-X-c_i \end{cases} \tag{31}$$

As this function is strictly increasing and continuous, it can be inverted to obtain the aggregate spot sales of firm i's competitors, $Y_{-i}(Y,X)$, compatible with the optimization behaviour of firm i, as a response to Y:

$$Y_{-i}(Y,X)=\begin{cases} 2Y-(a-X-c_i) & \text{if } Y<a-X-c_i \\ Y & \text{if } Y\geq a-X-c_i \end{cases} \tag{32}$$

Using $Y_{-i}(Y,X)=Y-y_i$, solve for y_i, which yields the best response of firm i to the industry spot sales Y. We denote this expression $y_i(Y,X)$:

$$y_i(Y,X)=\begin{cases} a-X-c_i-Y & \text{if } Y<a-X-c_i \\ 0 & \text{if } Y\geq a-X-c_i \end{cases} \tag{33}$$

As marginal costs are ordered, thresholds at which firms are active on the spot market are ordered: $a-X-c_N<...<a-X-c_i<...<...a-X-c_1$ so that it is possible to find the equilibrium aggregate spot sales by solving $Y=\sum_{i=1}^{N} y_i(Y,X)$ for each possible value of X. However, the number of active firms must be determined in order to do this. Let k be the number of active firms, so firms $1,...,k$ have positive spot sales and firms $k+1,...,N$ have zero spot sales. Using (33) total spot sales, denoted $Y_k^*(X)$, solves $Y_k^*(X)=\sum_{i=1}^{k} y_i(Y_k^*(X),X)$, giving

$$Y_k^*(X)=\frac{k(a-X)-k\bar{c}_k}{k+1}. \tag{34}$$

For this to be an equilibrium it must be the case that $Y_k^*(X)<a-X-c_k$, so that firms $1,...,k$ are active, and that $Y_k^*(X)\geq a-X-c_{k+1}$, so that firms $k+1,...,N$ are inactive. So, for active firms, this requires $X\leq a+k\bar{c}_k-(k+1)c_k=X_k^d$, whereas for inactive firms it requires $X\geq a+k\bar{c}_k-(k+1)c_{k+1}=X_{k+1}^d$. Hence, firm $i=1,2,...,N$ is active in equilibrium if, and only if, $X\leq X_i^d=a+i\bar{c}_i-(i+1)c_{i+1}$, where $X_1^d>X_2^d>...>X_N^d$.

In the sub-game defined by X, firm i is active if $X<X_i^d$ and, since the X_i^d are decreasing in i, the number of active firms, $n(X)$, is simply the number of firms for which $X<X_k^d$, or

$$n(X)=\sum_{j=1}^{N}\mathbb{1}_{[X<X_k^d]}. \tag{35}$$

For active firms we have

$$\begin{aligned}y_i^*(X)&=y_i(Y_{n(X)}^*(X),X)\\&=a-X-c_i-\frac{n(X)}{n(X)+1}(a-n(X)\bar{c}_{n(X)}-X)\\&=\frac{a-X+n(X)\bar{c}_{n(X)}}{n(X)+1}-c_i\end{aligned} \tag{36}$$

and equilibrium price is

$$\begin{aligned}p_S^*(X)&=a-X-Y_{n(X)}^*(X)\\&=a-X-\frac{n(X)}{n(X)+1}(a-X-n(X)\bar{c}_{nX)})\\&=\frac{a-X+n(X)\bar{c}_{n(X)}}{n(X)+1}.\end{aligned} \tag{37}$$

To establish continuity of spot sales at the threshold X_n^d, where the number of firms changes, we have

$$\begin{aligned}\lim_{X\to X_n^{d-}}y_i^*(X)&=\frac{a-X_n^d+n\bar{c}_n}{n+1}-c_i\\&=\frac{a-(a+n\bar{c}_n-(n+1)c_n)+n\bar{c}_n}{n+1}-c_i\\&=c_n-c_i,\end{aligned} \tag{38}$$

and

$$\begin{aligned}\lim_{X\to X_n^{d+}}y_i^*(X)&=\frac{a-X_n^d+(n-1)\bar{c}_{n-1}}{n}-c_i\\&=\frac{a-(a+n\bar{c}_n-(n+1)c_n)+(n-1)\bar{c}_{n-1}}{n}-c_i\\&=\frac{(n+1)c_n-n\bar{c}_n+(n-1)\bar{c}_{n-1}}{n}-c_i\\&=\frac{nc_n+c_n-\sum_{j=1}^{n}c_j+\sum_{j=1}^{n-1}c_j}{n}-c_i\\&=c_n-c_i.\end{aligned} \tag{39}$$

Therefore, spot sales are continuous at the threshold X_n^d, and, consequently, so is price.

□

Proof of Lemma 1. At the points of discontinuity in firm i's marginal profit, (17), $X_k^d-X_{-i}$ for $k>i$, we have $p_S^*(X_k^d)=c_k$, where there are k active firms for x_i slightly below

$X_k^d{-}X_{-i}$ and $k{-}1$ active firms for x_i slightly above $X_k^d{-}X_{-i}$. Examining the limit of (17) as x_i approaches $X_k^d{-}X_{-i}$ from below and above we have

$$\lim_{x_i \to X_k^d - X_{-i}{}^-} \frac{\partial \pi_i(x_i, X_{-i})}{\partial x_i} = (c_k - c_i)\frac{k-1}{k+1} - \frac{X_k^d - X_{-i}}{k+1} \tag{40}$$

and

$$\lim_{x_i \to X_k^d - X_{-i}{}^+} \frac{\partial \pi_i(x_i, X_{-i})}{\partial x_i} = (c_k - c_i)\frac{k-2}{k} - \frac{X_k^d - X_{-i}}{k}. \tag{41}$$

Since $\frac{k-2}{k}<\frac{k-1}{k+1}$, $\frac{-1}{k}<\frac{-1}{k+1}$, and $c_k{-}c_i{>}0$, we have

$$\lim_{x_i \to X_k^d - X_{-i}{}^-} \frac{\partial \pi_i(x_i, X_{-i})}{\partial x_i} > \lim_{x_i \to X_k^d - X_{-i}{}^+} \frac{\partial \pi_i(x_i, X_{-i})}{\partial x_i}, \tag{42}$$

so marginal profit jumps down at $x_i{=}X_k^d{-}X_{-i}$. □

Proof of Lemma 2.

We start with the best response for firm N, $r_N(X_{-N})$, which is the simplest because there are no higher-cost rival firms to deter, so the marginal profit for firm N is continuous at any $x_N{\geq}0$. Using (17) and $n(X){=}N$, equating its marginal profit to zero provides a candidate best-response

$$x_N = \frac{N-1}{2N}\left(a + N\bar{c}_N - (N+1)c_N - X_{-N}\right), \tag{43}$$

which is strictly positive if

$$X_{-N} < a + N\bar{c}_N - (N+1)c_N = X_N^d. \tag{44}$$

If this condition on X_{-N} does not hold, firm N's profit is negative for any positive sales. Therefore, when $X_{-N}{\geq}X_N^d$, firm N is better off not selling in advance, $x_N{=}0$. To summarize,

$$r_N(X_{-i}) = \begin{cases} \frac{N-1}{2N}\left(a + N\bar{c}_N - (N+1)c_N - X_{-N}\right) & \text{if } X_{-N} < X_N^d \\ 0 & \text{if } X_{-N} \geq X_N^d. \end{cases} \tag{45}$$

A similar approach is used to find the best response functions of the other firms, however, these are complicated by the discontinuity in marginal profit at the activity thresholds of higher cost rivals. Suppose that X_{-i} is such that firm i is considering a level of advance sales consistent with $k{>}i$ active firms, i.e. $n(X){=}k$. One of two possibilities must be true: either i) firm i's best response occurs on the downward sloping part of its marginal profit, between $X_{k+1}^d{-}X_{-i}$ and $X_k^d{-}X_{-i}$, or ii) firm i's best response occurs at $X_{k+1}^d{-}X_{-i}$, where i's marginal profit jumps from positive to negative. We will examine each case in turn, deriving the bounds on X_{-i} under which they obtain.

Case i) $r_i(X_{-i}){\in}(X_{k+1}^d{-}X_{-i}, X_k^d{-}X_{-i})$: In this case, i's best-response is

$$r_i(X_{-i}) = r_{i,k}(X_{-i}) = \frac{k-1}{2k}\left(a + k\bar{c}_k - (k+1)c_i - X_{-i}\right), \tag{46}$$

which is in the interval $(X^d_{k+1}-X_{-i}, X^d_k-X_{-i})$ if marginal profit is positive at the lower limit of the interval and negative at the upper limit:

$$\lim_{x_i \to X^d_{k+1}-X_{-i}{}^+} \frac{\partial \pi_i(x_i, X_{-i})}{\partial x_i} > 0 \qquad \text{and} \qquad \lim_{x_i \to X^d_k-X_{-i}{}^-} \frac{\partial \pi_i(x_i, X_{-i})}{\partial x_i} < 0. \tag{47}$$

This requires

$$\lim_{x_i \to X^d_{k+1}-X_{-i}{}^+} \frac{\partial \pi_i(x_i, X_{-i})}{\partial x_i} = (c_{k+1}-c_i)\frac{k-1}{k+1} - \frac{X^d_k - X_{-i}}{k+1} > 0, \tag{48}$$

which reduces to

$$X_{-i} > X^d_{k+1} - (k-1)(c_{k+1}-c_i). \tag{49}$$

For the upper bound,

$$\lim_{x_i \to X^d_k-X_{-i}{}^-} \frac{\partial \pi_i(x_i, X_{-i})}{\partial x_i} = (c_k-c_i)\frac{k-1}{k+1} - \frac{X^d_k - X_{-i}}{k+1} < 0, \tag{50}$$

which reduces to

$$X_{-i} < X^d_k - (k-1)(c_k-c_i). \tag{51}$$

Consequently, (46) is firm i's best response to X_{-i} if

$$X^d_{k+1} - (k-1)(c_{k+1}-c_i) < X_{-i} < X^d_k - (k-1)(c_k-c_i). \tag{52}$$

This range is clearly not empty since $X^d_{k+1} < X^d_k$ and $c_{k+1} > c_k$.

Case ii) $r_i(X_{-i}) = X^d_{k+1} - X_{-i}$: This requires that i's marginal profit jumps downward from positive to negative at $x_i = X^d_{k+1} - X_{-i}$, or

$$\lim_{x_i \to X^d_{k+1}-X_{-i}{}^+} \frac{\partial \pi_i(x_i, X_{-i})}{\partial x_i} < 0 \qquad \text{and} \qquad \lim_{x_i \to X^d_{k+1}-X_{-i}{}^-} \frac{\partial \pi_i(x_i, X_{-i})}{\partial x_i} > 0. \tag{53}$$

The first condition, immediate from (49), is

$$X_{-i} < X^d_{k+1} - (k-1)(c_{k+1}-c_i), \tag{54}$$

while the second condition requires

$$\lim_{x_i \to X^d_{k+1}-X_{-i}{}^-} \frac{\partial \pi_i(x_i, X_{-i})}{\partial x_i} = (c_{k+1}-c_i)\frac{k}{k+2} - \frac{X^d_{k+1} - X_{-i}}{k+2} > 0, \tag{55}$$

which reduces to

$$X_{-i} > X^d_{k+1} - k(c_{k+1}-c_i). \tag{56}$$

So $X^d_{k+1} - X_{-i}$ is firm i's best response to X_{-i} if

$$X^d_{k+1} - k(c_{k+1}-c_i) < X_{-i} < X^d_{k+1} - (k-1)(c_{k+1}-c_i), \tag{57}$$

which is a non-empty interval for $c_i < c_{k+1}$.

The general form of the best-response function for firms $i{=}2,...,N$ is then

$$r_i(X_{-i}){=}\begin{cases} r_{i,N}(X_{-i}), & \text{if } X_{-i}{\le}X_N^d{-}(N{-}1)(c_N{-}c_i),\\ X_N^d{-}X_{-i} & \text{if } X_N^d{-}(N{-}1)(c_N{-}c_i){<}X_{-i}{\le}X_N^d{-}(N{-}2)(c_N{-}c_i),\\ ... & \\ r_{i,k}(X_{-i}) & \text{if } X_{k+1}^d{-}(k{-}1)(c_{k+1}{-}c_i){<}X_{-i}{\le}X_k^d{-}(k{-}1)(c_k{-}c_i),\\ X_k^d{-}X_{-i} & \text{if } X_k^d{-}(k{-}1)(c_k{-}c_i){<}X_{-i}{\le}X_k^d{-}(k{-}2)(c_k{-}c_i),\\ ... & \\ r_{i,i}(X_{-i}) & \text{if } X_{i+1}^d{-}(i{-}2)(c_{i+1}{-}c_i){<}X_{-i}{<}X_i^d\\ 0 & \text{if } X_{-i}{\ge}X_i^d. \end{cases} \tag{58}$$

Each component of the best-response in (58) is clearly downward-sloping, and since $r_{i,k}(X_k^d{-}(k{-}1)(c_k{-}c_i)){=}(k{-}1)(c_k{-}c_i)$ and $r_{i,k-1}(X_k^d{-}(k{-}2)(c_k{-}c_i)){=}(k{-}2)(c_k{-}c_i)$, the best-response functions are also continuous.

□

Proof of Proposition 2.

We apply the same method used in the proof of Proposition A, add X_{-i} to firm i's best-response, (58), determined in the proof of Lemma 2. However, the derivation of the inclusive best-response, $\tilde{r}_i(X)$, from $r_i(X_{-i})$ is complicated in this case due to the possibility of the equilibrium being a deterrence one. We show below that this possibility results in $\tilde{r}_i(X)$ being a correspondence rather than a function.

We first consider possible equilibria with at least two active firms. The possibility of a monopoly for firm one is considered separately below. Add X_{-i} to the individual best response we obtain the industry sales X compatible with firm i's optimization, which we denote $X^{(i)}(X_{-i})$:

$$X^{(i)}(X_{-i}){=}\begin{cases} \frac{N-1}{2N}(a{+}N\bar{c}_N{-}(N{+}1)c_i){+}\frac{N+1}{2N}X_{-i}, & \text{if } X_{-i}{\le}X_N^d{-}(N{-}1)(c_N{-}c_i),\\ X_N^d & \text{if } X_N^d{-}(N{-}1)(c_N{-}c_i){<}X_{-i}{\le}X_N^d{-}(N{-}2)(c_N{-}c_i),\\ ... & \\ \frac{k-1}{2k}(a{+}k\bar{c}_k{-}(k{+}1)c_i){+}\frac{k+1}{2k}X_{-i}, & \text{if } X_{k+1}^d{-}(k{-}1)(c_{k+1}{-}c_i){<}X_{-i}{\le}X_k^d{-}(k{-}1)(c_k{-}c_i),\\ X_k^d & \text{if } X_k^d{-}(k{-}1)(c_k{-}c_i){<}X_{-i}{\le}X_k^d{-}(k{-}2)(c_k{-}c_i),\\ ... & \\ \frac{i-1}{2i}(a{+}i\bar{c}_i{-}(i{+}1)c_i){+}\frac{i+1}{2i}X_{-i}, & \text{if } X_{i+1}^d{-}(i{-}2)(c_{i+1}{-}c_i){<}X_{-i}{<}X_i^d\\ X_{-i} & \text{if } X_{-i}{\ge}X_i^d. \end{cases} \tag{59}$$

As each $r_i(X_{-i})$ is continuous, the function $X^{(i)}(X_{-i})$ is also continuous. However, it is only weakly increasing, since it is constant for $X_k^d{-}(k{-}1)(c_k{-}c_i){<}X_{-i}{\le}X_k^d{-}(k{-}2)(c_k{-}c_i)$.

In order to determine the inclusive best reply, we need to invert $X^{(i)}(X_{-i})$ to obtain the advance sales of i's rivals that generate aggregate sales of X when firm i plays its best-response. Since $X^{(i)}(X_{-i})$ is equal to a constant X_k^d for $k{=}i{+}1,...,N{-}1$, when deterrence occurs, there is a range of values for X_{-i} that result in $X{=}X_k^d$. In particular, for $X_k^d-(k-1)(c_k-c_i)<X_{-i}<X_k^d-(k-2)(c_k-c_i)$ the inverse of the function $X^{(i)}(X_{-i})$ is a correspondence. We let $X_{-i}^{(i)}(X)$ denote this correspondence:

$$X_{-i}^{(i)}(X)=\begin{cases}\frac{2N}{N+1}X-\frac{N-1}{N+1}(a+N\bar{c}_N-(N+1)c_i), & \text{if } X<X_N^d,\\ \in(X_N^d-(N-1)(c_N-c_i),X_N^d-(N-2)(c_N-c_i)), & \text{if } X=X_N^d,\\ \ldots & \\ \frac{2k}{k+1}X-\frac{k-1}{k+1}(a+k\bar{c}_k-(k+1)c_i), & \text{if } X_{k+1}^d<X<X_k^d,\\ \in(X_k^d-(k-1)(c_k-c_i),X_k^d-(k-2)(c_k-c_i)), & \text{if } X=X_k^d,\\ \ldots & \\ \frac{2i}{i+1}X-\frac{i-1}{i+1}(a+i\bar{c}_i-(i+1)c_i), & \text{if } X_{i+1}^d<X<X_i^d\\ X_{-i} & \text{if } X\geq X_i^d.\end{cases} \tag{60}$$

We now can solve for x_i in $X-x_i=X_{-i}^{(i)}(X)$ to obtain the inclusive best-response, $\tilde{r}_i(X)$:

$$\tilde{r}_i(X)=\begin{cases}\frac{N-1}{N+1}(a+N\bar{c}_N-(N+1)c_i)-\frac{N-1}{N+1}X, & \text{if } X<X_N^d,\\ \in((N-2)(c_N-c_i),(N-1)(c_N-c_i)), & \text{if } X=X_N^d,\\ \ldots & \\ \frac{k-1}{k+1}(a+k\bar{c}_k-(k+1)c_i)-\frac{k-1}{k+1}X, & \text{if } X_{k+1}^d<X<X_k^d,\\ \in((k-2)(c_k-c_i),(k-1)(c_k-c_i)), & \text{if } X=X_k^d,\\ \ldots & \\ \frac{i-1}{i+1}(a+i\bar{c}_i-(i+1)c_i)-\frac{i-1}{i+1}X, & \text{if } X_{i+1}^d<X<X_i^d\\ 0 & \text{if } X\geq X_i^d.\end{cases} \tag{61}$$

The general procedure for finding the equilibrium to aggregative games would now sum the individual inclusive best-responses and solve for the equilibrium X. It is not quite so straightforward in this case due to the inclusive best-responses being correspondences. However, since the condition for individual firms to choose a deterrence level of advance sales is not firm-specific ($X=X_k^d$), we can aggregate the inclusive best-response correspondences to form an aggregate inclusive best-response function. Either the aggregate inclusive best-response is corresponds to a non-deterrence level of advance sales, $\sum_{i=1}^k \tilde{r}_i(X)\in(X_{k+1}^d,X_k^d)$, or it corresponds to a deterrence level of advance sales, $\sum_{i=1}^k \tilde{r}_i(X)=X_{k+1}^d$. We examine these two possibilities in turn, deriving the conditions on model parameters under which they obtain.

If $X^*\in(X_{k+1}^d,X_k^d)$, aggregating (61) yields

$$X^*=\frac{k(k-1)}{k+1}(a-\bar{c}_k-X^*) \quad\Longrightarrow\quad X^*=\frac{k(k-1)}{k^2+1}(a-\bar{c}_k). \tag{62}$$

For this to be an equilibrium it must lie in the interval (X_{k+1}^d,X_k^d). For $X^*<X_k^d$ we require

$$\frac{k(k-1)}{k^2+1}(a-\bar{c}_k)<a+k\bar{c}_k-(k+1)c_k, \tag{63}$$

which can be expressed as

$$a>c_k+k^2(c_k-\bar{c}_k)\equiv\alpha_k. \tag{64}$$

For $X^*{>}X^d_{k+1}$ we require

$$\frac{k(k-1)}{k^2+1}(a-\bar{c}_k)>a-(k+2)c_{k+1}+(k+1), \tag{65}$$

which reduces to

$$a<c_{k+1}+k^2(c_{k+1}-\bar{c}_k)\equiv\alpha^d_k. \tag{66}$$

Hence, the equilibrium is of the non-deterrence type with k active firms if $\alpha_k<a<\alpha^d_k$. To allow this statement to be applied for $k{=}N$, we define $\alpha^d_N{=}\infty$, since if N firms are active, there is no other firm to deter.

Notice that the activity threshold level of demand for firm $k{+}1$ is strictly higher than the deterrence threshold for firm $k{+}1$: $\alpha_{k+1}{=}c_{k+1}+(k+1)^2(c_{k+1}-\bar{c}_{k+1})>c_{k+1}+k^2(c_{k+1}-\bar{c}_k){=}\alpha^d_k$. For $a\in[\alpha^d_k,\alpha_{k+1}]$, the equilibrium is a deterrence one, with k active firms producing X^d_{k+1} in aggregate. Although aggregate advance sales are uniquely determined in this case, individual advance sales are not. There are a continuum of equilibrium advance sales vectors that must satisfy $x^*_i{=}((k-2)(c_k-c_i),(k-1)(c_k-c_i))$ and $X^*{=}X^d_{k+1}{=}\sum_{i=1}^k x^*_i$.

Finally, consider the possibility that the equilibrium has only one active firm, firm one. From the inclusive best reply, it should be noted that when $X{>}X^d_2$, $x_1(X){=}0$.[25] Since $X{=}0$ in this case, it can only be an equilibrium if $X^d_2{<}0$, or $a<2c_2-c_1{=}\alpha^d_1$, in which case firm two's activity is blockaded. We define $\alpha_1{=}c_1$, so this case occurs for $a\in(\alpha_1,\alpha^d_1)$, firm one is active on the spot market even though it is not on the advance sales market. For $a\in[\alpha^d_1,\alpha_2]$, firm one chooses $x^*_1{=}X^d_2$, deterring the activity of firm two.

In summary, given $(c_1,c_2,...,c_N)$, there are threshold levels of demand, $\alpha_1<\alpha^d_1<...<\alpha^d_{N-1}<\alpha_N<\alpha^d_N{=}\infty$, for which the number of active firms in equilibrium is given by $n^*{=}\max\{k|a{>}\alpha_k\}$. The equilibrium is a non-deterrence one if $\alpha_{n^*}\le a<\alpha^d_{n^*}$ and a deterrence one if $\alpha^d_{n^*}\le a<\alpha_{n^*+1}$). In each case, the aggregate level of advance sales, X^*, is unique, and, consequently, so is the equilibrium price $p^*_A{=}p^*_S(X^*)$.

□

Proof of Proposition 5.

a) For $a\in(\alpha_n,\alpha^d_n]$, Proposition 2 establishes that aggregate advance sales are $X^{nd}(n){=}\frac{n(n-1)}{n^2+1}(a-\bar{c}_n)$. Using the inclusive best-response, (61), we have $x^{nd}_i(n){=}\tilde{r}_i(X^{nd}(n))$ we have

$$\begin{aligned} x^{nd}_i(n)&=\frac{n-1}{n+1}\left(a+n\bar{c}-(n+1)c_i-\frac{n(n-1)}{n^2+1}(a-\bar{c})\right)\\ &=\frac{n-1}{n+1}\left(\frac{n+1}{n^2+1}a+\frac{n^2(n+1)}{n^2+1}\bar{c}-(n+1)c_i\right)\\ &=(n-1)\left(\frac{a+n^2\bar{c}_n}{n^2+1}-c_i\right) \end{aligned}\tag{67}$$

[25] Note that the efficient firm is not indifferent among all combinations of advanced and spot sales adding up to the monopoly output in this case since any $x_1{>}0$ results in a price lower than the monopoly price as the firm cannot commit to not make additional spot sales at a lower price. This is essentially the same problem that is faced by a durable goods monopolist.

and price $p_A^{nd}(n)=p_S^*(X^{nd}(n))$ or

$$
\begin{aligned}
p_A^{nd}(n) &= \frac{a+n\bar{c}_n - X^{nd}(n)}{n+1}, \\
&= \frac{a+n\bar{c}_n - \frac{n(n-1)}{n^2+1}(a-\bar{c}_n)}{n+1}, \\
&= \frac{1}{n+1}\left(\frac{n+1}{n^2+1}a+\frac{n^2(n+1)}{n^2+1}\bar{c}_n\right), \\
&= \frac{a+n^2\bar{c}_n}{n^2+1}.
\end{aligned} \tag{68}
$$

b) For $a\in(\alpha_n^d, \alpha_{n+1}]$, Proposition (2) establishes that there is a deterrence equilibrium with aggregate advances sales of X_{n+1}^d, and, consequently, an advance price of $p_A^*=c_{n+1}$. From the best response functions given in (58) in the proof of Lemma 2, firm $i\leq n$ will choose the level of sales that deters firm $n+1$ if

$$X_{n+1}^d-(n)(c_{n+1}-c_i)<X_{-i}<X_{n+1}^d-(n-1)(c_{n+1}-c_i). \tag{69}$$

The maximum sales that firm i can have in a deterrence equilibrium occurs when X_{-i} is at the lower bound of this range:

$$x_i^{max}=X_{n+1}^d-(X_{n+1}^d-(n)(c_{n+1}-c_i))=n(c_{n+1}-c_i). \tag{70}$$

The minimum sales that firm i can have in a deterrence equilibrium occurs when X_{-i} is at the upper bound of this range:

$$x_i^{min}=X_{n+1}^d-(X_{n+1}^d-(n-1)(c_{n+1}-c_i))=(n-1)(c_{n+1}-c_i). \tag{71}$$

□

B The effect of deterrence on price

The maximum price reduction that is attributable to deterrence of firm $n+1$ is presented in (22), which we can express as a proportion of the deterrence price as

$$\frac{p_A^{nd}(n)-c_{n+1}}{c_{n+1}}=\frac{n}{n^2+1}\left(1-\frac{\bar{c}_n}{c_{n+1}}\right). \tag{72}$$

This depends only on the number of active firms, and the marginal cost of the deterred firm relative to the average marginal cost of active firms. We tabulated the percentage price effect for a variety of values of n and $\bar{c}_n/c_{n+1}$ in Table 2.

n vs $\bar{c}_n/c_{n+1}$	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%
1	25%	23%	20%	18%	15%	13%	10%	8%	5%	3%
2	20%	18%	16%	14%	12%	10%	8%	6%	4%	2%
3	15%	14%	12%	11%	9%	8%	6%	5%	3%	2%
4	12%	11%	9%	8%	7%	6%	5%	4%	2%	1%
5	10%	9%	8%	7%	6%	5%	4%	3%	2%	1%
6	8%	7%	6%	6%	5%	4%	3%	2%	2%	1%
7	7%	6%	6%	5%	4%	4%	3%	2%	1%	1%
8	6%	6%	5%	4%	4%	3%	2%	2%	1%	1%
9	5%	5%	4%	4%	3%	3%	2%	2%	1%	1%
10	5%	4%	4%	3%	3%	2%	2%	1%	1%	0%
11	5%	4%	4%	3%	3%	2%	2%	1%	1%	0%
12	4%	4%	3%	3%	2%	2%	2%	1%	1%	0%
13	4%	3%	3%	3%	2%	2%	2%	1%	1%	0%
14	4%	3%	3%	2%	2%	2%	1%	1%	1%	0%
15	3%	3%	3%	2%	2%	2%	1%	1%	1%	0%
16	3%	3%	2%	2%	2%	2%	1%	1%	1%	0%
17	3%	3%	2%	2%	2%	1%	1%	1%	1%	0%
18	3%	2%	2%	2%	2%	1%	1%	1%	1%	0%
19	3%	2%	2%	2%	2%	1%	1%	1%	1%	0%
20	2%	2%	2%	2%	1%	1%	1%	1%	0%	0%

Table 2: Maximal price variation when n and $\bar{c}_n/c_{n+1}$ vary

C The most- and least-efficient deterrence equilibria

In a deterrence equilibrium with n active firms, the **most-efficient deterrence equilibrium** has individual advance sales concentrated as much as possible in efficient firms. This means that there is a "marginal" firm, firm m, for which sales of all firms $i<m$ are at their maximum defined in Proposition 5b), and sales of all firms $j>m$ are at their minimum defined in Proposition 5b). The marginal firm then sells the residual amount necessary for aggregate sales to equal the required deterrence level, X_n^d. Formally,

$$x_i^* = \begin{cases} n(c_{n+1} - c_i), & \text{for } i<m, \\ X_n^d - \sum_{j \neq m} x_j^*, & \text{for } i=m, \\ (n-1)(c_{n+1} - c_i), & \text{for } i>m. \end{cases} \tag{73}$$

The **least-efficient deterrence equilibrium** is determined similarly, except that the marginal firm, m' (not necessarily equal to m above), is defined by all firms more efficient than m' sell at their minimum and all firms less efficient than m' sell at their maximum:

$$x_i^* = \begin{cases} (n-1)(c_{n+1} - c_i), & \text{for } i<m', \\ X_n^d - \sum_{j \neq m'} x_j^*, & \text{for } i=m', \\ n(c_{n+1} - c_i), & \text{for } i>m'. \end{cases} \tag{74}$$

In each case, the marginal firm, m or m', is uniquely determined by X_n^d, and, hence, depends on the level of $a \in (\alpha_n^d, \alpha^n]$.