Speculative storage in imperfectly competitive markets

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Abstract

Markets for many commodities are characterized by imperfectly competitive production as well as substantial storage by speculators who are attracted by significant price volatility. We examine how speculative storage affects the behaviour of an oligopoly producing a commodity for which demand is subject to random shocks. Speculators compete with consumers when purchasing the commodity and then subsequently compete with producers when selling their stocks, resulting in two opposing incentives: on the one hand, producers would like to increase production to capture future sales in advance by selling to speculators; while on the other hand, they would like to withhold production to deter speculation, thereby eliminating the additional supply from speculators in future periods. We find that the incentive to sell to speculators can be quite strong, potentially resulting in prices sufficiently high to drive consumers from the market. Furthermore, these incentives are non-monotonic in the number of producers: speculative storage occurs more frequently in a relatively concentrated oligopoly than in the extremes of monopoly or perfect competition.

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1 Introduction

Commodity market speculation and imperfect competition are two topics that have long been of interest to economists. Despite the attention given to each topic individually, little is known about market performance in circumstances where oligopolistic producers and competitive speculators interact, a topic that has recently been the subject of considerable public attention.[1] This attention has triggered substantial academic interest investigating the relationship between inventories and speculative trading on commodity markets from an econometric point of view.[2] However, little is known from a theoretical perspective on the performance of these markets in which oligopolistic producers face the constraints that competitive speculation places on price dynamics. In this setting, strategic considerations may be fundamental to production decisions which ultimately affect the level of inventories carried in equilibrium. Understanding the consequences of the interaction between imperfect competition and competitive speculation is the issue addressed in this paper.

We analyze a model in which an oligopoly produces a good over a finite time horizon with demand subject to stochastic shifts. In addition to producers and consumers there are speculators who are able to store the good between periods. When storage by these speculators is possible, the net demand faced by producers is affected: on the one hand, speculators increase demand for the product when price is low as they buy the good to store for future sale; on the other hand, speculators increase the supply of the product when price is high and they wish to sell from their inventories. Hence speculators are competitors to producers when selling and competitors to consumers when buying, changing the effective demand faced by producers. With imperfectly competitive producers, this effect of storage on net demand raises the possibility that producers' behaviour may be affected through strategic motives to influence future demand states. It is the main purpose of this paper to analyze the strategic effect that speculative storage has on producer behaviour.

We obtain the result that, when expected demand growth is relatively large, speculators may purchase the entire first period output, resulting in zero first period consumption. This occurs when speculators value the good in the first period more highly than consumers do essentially out-bidding them for the current production.[3] This result is particularly interesting

[1] For example, the impact of shortage risks on some strategic commodity markets has been the object of the European Commission (2011), who list 14 critical raw materials (Antimony, Beryllium, Cobalt, Fluorspar, Gallium, Germanium, Graphite, Indium, Magnesium, Niobium, Platinum Group Metals, Rare earths, Tantalum, and Tungsten) for which concentration is high and supply lies in the hands of a few firms or in the control of a very small number of countries. Smith (2009) points out that the world oil market is characterized by oligopolistic supply where OPEC is dominant along with speculation by both "commercial" and "non-commercial" traders. Finally, non-ferrous minerals such as nickel and aluminum are produced in oligopolistic conditions and trade on the London Metal Exchange which facilitates speculation.

[2] Frankel and Rose (2010), Kilian (2008, 2009), Kilian and Murphy (2014).

[3] This equilibrium is related to the "entitlement failure" used by Sen (1981) to explain how famines can occur even when production is not particularly low and storehouses contain grain. In this situation, speculators'

since in much of the public discussion of recent events in commodity markets there is a recognition, implicit or explicit, of the extreme form of competition speculators exert to other market participants, a competition that is often viewed as detrimental by neither "ensuring an efficient, economic and regular supply to consumers" nor "a fair return for those investing in the industry"[4]. In this respect, the example of developing countries struggling to satisfy their needs on the demand side, as emphasized by regulators or international agencies, is striking. In circumstances where current demand is low compared to expected future demand, which could correspond to the recent economic boom in Asia leading to expectations of rapid growth in demand, the entire output may be purchased without being consumed and paid at the expected future price which is disconnected from current demand. In that case, the market price is "high", the inventories are "large", and the consumption is "nil".[5] Our analysis demonstrates that the likelihood that we see this outcome is increasing as production becomes more concentrated.

We also find that the extent to which equilibria arise in which speculators carry stocks is non-monotonic in the number of firms, being lower for the two extremes of monopoly and perfect competition and highest for an intermediate number of firms. This result is due to two countervailing forces in our model: first, producers have an incentive to sell to speculators in order to capture future sales in advance; second, producers have an incentive to deter speculation in order to limit the future competition that is represented by speculative inventories. While the former incentive is absent in monopoly, but present in every oligopolistic market where a price differential allows speculators to store, the latter exists only when the price differential is limited, so that a small reduction of output reduces speculation. This may happen either under monopoly, where the firm reaps the entire benefit of deterring speculation, or under more competitive market structures, where prices are closer to marginal cost which limits the benefit of sales to speculators.

Modern approaches to the study of speculation in perfectly competitive markets was pioneered by Gustafson (1958) who was the first to formulate the problem in a dynamic programming context and highlight the difficulty for the solution of the model caused by the restriction that aggregate inventories cannot be negative. Samuelson (1971) showed that the speculative storage problem could be reformulated as a social planner's problem. The theory pertaining to the rational expectations competitive storage model was further developed in a series of works including Newbery and Stiglitz (1981), Scheinkman and Schechtman (1983), Newbery (1984), Williams and Wright (1991), Deaton and Laroque (1992), and McLaren (1999). The

willingness to pay for the available supply exceeds that of consumers. This case is also relevant to the policy debate regarding anti-hoarding laws (see Wright and Williams (1984)).

[4]These objectives are part of the OPEC mission in the petroleum industry, as can be read on their website, http://www.opec.org/opec_web/en/about_us/23.htm

[5]Even without the full exclusion of consumption, when both consumers and speculators are purchasing output, current consumption is displaced by speculative purchases to some extent.

focus in this literature is on the effects of storage on the distribution of prices caused by the movement of production across periods resulting from random production (harvest) shocks. As aggregate inventories cannot be negative, speculators smooth prices across periods only when positive inventories exist. Unexpectedly high prices result in stock-outs which leads to a breakdown of the price smoothing role of speculative storage.

Market power in commodity markets has been considered by examining imperfect competition in the storage function (Newbery (1984), Williams and Wright (1991), and McLaren (1999)), but production itself remains perfectly competitive in these works, hence there is no scope for strategic considerations on the part of producers. Another common element of these works is a focus on agricultural commodities, for which the assumption of perfectly competitive production is justifiable. However, for other commodities, such as metals, the assumption of perfectly competitive production is harder to justify. The problem of a monopoly producer facing competitive storage was touched on briefly in Newbery (1984), who demonstrated that speculation resulted in discontinuous marginal revenue for the monopolist. Mitraille and Thille (2009) examine a monopolist producer facing competitive storage using a model with capacity constrained speculators. They show that both the level and variance of prices are affected by speculation under monopolistic production. In particular, the presence of speculators may cause the monopolist to set a price higher than the monopoly one in order to eliminate competition from speculative sales in future periods, a result similar to that of limit pricing. In this paper we allow for an arbitrary number of firms and impose no constraint on speculative capacity, which allows us to examine the consequences of speculation for the functioning of an oligopolistic market.

Our paper lies at the cross-roads of two streams of research in industrial organization: imperfect competition under demand uncertainty on the one hand, and the literature studying strategic uses of storage on the other hand. Demand uncertainty was examined in the seminal article by Klemperer and Meyer (1986), in which price or quantity competition interacts with demand uncertainty in a static game where decisions are taken before uncertainty is revealed in order to examine the preference of firms for price versus quantity competition. In our dynamic model of Cournot competition, demand uncertainty can cause producers to manipulate speculation in either a pro- or anti-competitive manner, depending on current demand relative to future demand.

The strategic use of storage has been examined in cases where either producers or consumers perform the storage. Arvan (1985), Thille (2006), and Mitraille and Moreaux (2013) examine the former case, in which producers store for strategic reasons, as inventories are accumulated in order to gain a cost advantage. Storage by consumers is more similar to the model of speculative storage that we examine in that storage is undertaken by competitive agents. Consumer storage with duopoly production is examined in Anton and Varma (2005) and Guo and Villas-Boas (2007). Whereas Anton and Das Varma show that consumer storage has a pro-competitive effect in a two-period duopoly, Guo and Villas-Boas show that horizontal differentiation may mitigate this effect. Consumer storage under monopoly production is examined by Dudine et al. (2006) who find that consumer storage can have an anti-competitive effect as the monopolist may raise price to limit storage. By examining a general n-firm oligopoly facing demand uncertainty, we are able to discuss under what conditions these pro- and anti-competitive effects occur. Furthermore, we show that the effect of speculation on the outcome of an oligopolistic market is non-monotonic in the number of competitors.

We next provide a description of our T-period model, followed by an analysis of the equilibrium in period $T-1$, for which we obtain some analytic results due to the fact that the period T equilibrium is relatively straightforward to obtain. We then demonstrate the robustness of these results by computing the equilibrium for an earlier period in which the influence of the final period is minimal.

2 Model

Consider the market for a homogeneous product consisting of producers, consumers, and speculators. We wish to focus on the effects of speculation alone, so we assume that all agents are risk-neutral. There are n producers who compete in quantities over T periods. Let $\delta \in (0,1]$ be the agents' common discount factor. In each period t, firm $i \in \{1,...,n\}$ chooses the quantity to produce, $q_t^i \in \mathbb{R}^+$. All firms face the same quadratic cost function of $\frac{c}{2}{q_t^i}^2$. Convex production costs are not crucial to our results, however, as we discuss below, speculators' behaviour can lead to non-concavities in producer revenue, which can be mitigated by assuming sufficiently convex production costs. We assume that firms are unable to store their production between the two periods.[6]

In order for speculation to be potentially profitable, we require some price variation over time. We ensure this by having demand vary randomly across periods,[7] which also implies that storage is potentially socially valuable in this model as it allows producers to smooth their convex production costs over time. In each period, consumers' aggregate demand for the product is given by,

$$D(a_t, p_t) = \max[a_t - p_t, 0] \tag{1}$$

where a_t is drawn from a uniform distribution on $[0, A]$ for each t. Combined with sufficient cost convexity, the assumption of a uniform distribution for the demand uncertainty allows

[6] Clearly it would be more realistic to allow firms to also store, however that would add a substantial degree of complexity to the analysis potentially clouding the effects of independent speculators. See Arvan (1985), Thille (2006), and Mitraille and Moreaux (2013) for analyses of producer storage under imperfect competition.

[7] Having the source of variation be in demand means that for a sufficiently large number of producers speculation will not occur as price will tend to equality in the two periods.

us to compute closed-form solutions for the equilibrium quantities in period $T-1$. Random changes in a_t may be interpreted as random shocks affecting the distribution of income in the population of consumers from period to period, modifying the willingness to pay for the product sold by firms and stored by speculators. The distribution of the intercept of market demand is common knowledge to both producers and speculators who learn the current state of demand, a_t, prior to any production or storage decisions in a period. Information is consequently symmetric between producers and speculators.

We denote the aggregate quantity produced in period t by Q_t, and the aggregate quantity produced by all firms but i by Q_t^{-i}, where $Q_t = \sum_{i=1}^n q_t^i$ and $Q_t^{-i} = \sum_{j=1,j\neq i}^n q_t^j$. Producer i's payoff, Π^i, is

$$\Pi^i = E_0 \sum_{t=1}^{T} \delta^t [p_t q_t^i - \frac{c}{2} {q_t^i}^2], \tag{2}$$

where E_t denotes the expectation operator with regard to the random demand intercept conditional on information at time t.

We model speculation in the same way as is done in the competitive speculative storage literature (Newbery (1984), Deaton and Laroque (1992, 1996)). In each period a large number of risk-neutral, price-taking agents exist who have access to a storage technology and face no borrowing constraints. The storage technology allows speculators to store the commodity at a unit cost of w between periods, so the expected return to storing a unit of the commodity between periods t and $t+1$ is $\delta(E_t[p_{t+1}] - w) - p_t$. We use X_t to denote aggregate speculative sales in period t (purchases have $X_t < 0$) and H_t to denote beginning of period stocks. Consequently, inventory dynamics follow $H_{t+1} = H_t - X_t$ and we assume that inventories unsold at the end of period T can be disposed of at no cost. To ensure that speculation is at least potentially profitable, we assume that consumers expected valuation exceeds the unit cost of storage, $A/2 \geq w$. In addition, since it plays an important role in the model, we define $p_t^e(H_{t+1}) = E_t[p_{t+1}]$ to represent the expected next-period price when stocks of H_{t+1} are carried into the next period.

The timing of moves within a period is simply the standard Cournot timing with demand adjusted for net speculative supply: producers observe a_t and H_t and then set their output, q_t^i. Price adjusts to clear the market with demand now equal to the sum of consumer demand and speculators' net demand. We search for the subgame-perfect Nash equilibrium to this game with rational expectations on the part of all agents. In any period $t = 1, 2, ..., T$, $V_{i,t}(H_t, a_t)$ will denote the present value of the expected payoff to producer i from t to T when producers play their Nash equilibrium strategies, and $V_{i,t}^e(H_{t+1}) = E_t\big[V_{i,t+1}(H_{t+1}, a_{t+1})\big]$ will denote the expected continuation payoff to producer i when speculators carry H_{t+1} into the next period.

In order to complete the specification of producer payoffs, we now work out the effect of speculation on the net demand faced by producers. The aggregate behaviour of these speculators ensures that $p_t \geq \delta(p_t^e(H_{t+1}) - w)$ with a stock-out occurring if the inequality is strict. The non-negativity constraint on aggregate speculative inventories implies that speculators' aggregate behaviour satisfies the complementarity condition

$$H_t - X_t \geq 0, \quad p_t - \delta(p^e(H_{t+1}) - w) \geq 0, \quad (H_t - X_t)(p_t - \delta(p^e(H_{t+1}) - w)) = 0. \tag{3}$$

Either no inventories are carried and the return to storage is negative, or inventories are carried and the return to storage is zero. We will use $X_t^*(Q_t)$ to denote the equilibrium storage undertaken when producers sell Q_t in aggregate. The market clearing price, $P_t(Q_t)$, must be such that, given Q_t, the total of consumer and speculative purchases satisfy

$$D_t(a_t, P_t(Q_t)) - X_t^*(Q_t) = Q_t. \tag{4}$$

From (3), there is a threshold level of aggregate output, which we denote Q_t^L, below which $p_t > \delta(p_t^e(0) - w)$, as speculators cannot carry negative inventories. This threshold is the level of output which leads to zero return to speculation when there is a stockout:

$$a_t - H_t - Q_t^L = \delta(p_t^e(0) - w). \tag{5}$$

For $Q_t < Q_t^L$ the slope of net demand coincides with that of consumer demand, while for $Q_t > Q_t^L$ the slope of net demand differs from that of consumer demand due to the dependence of speculative sales on aggregate output. Note that it is possible that $Q_t^L < 0$ in which case a stockout cannot occur.

Another threshold for aggregate output is possible due to the competition that speculators pose to consumers when they purchase the good for storage. Speculators' purchase the good when the price is relatively low. To the extent that a low price results from a low a_t, speculator and consumer demands are negatively correlated in this situation, introducing the possibility that speculators squeeze consumers out of the market. The exclusion of consumers will occur if speculators value producers' output more highly than consumers do: $\delta(p_t^e(H_t + Q_t) - w) > a_t$. Consequently, there is another threshold output, which we denote $\widehat{Q}_t$, for which only speculators buy if $Q_t < \widehat{Q}_t$ and consumers buy and speculators carry inventories if $Q_t > \widehat{Q}_t$. This threshold is determined by the level of aggregate output that just extinguishes consumer demand:

$$a_t = \delta(p_t^e(H_t + \widehat{Q}_t) - w). \tag{6}$$

As with Q_t^L, the slope of the demand faced by producers changes discontinuously at $\widehat{Q}_t$. It is important to note that only one of Q_t^L and $\widehat{Q}_t$ can be positive as long as $p_t^e()$ is a decreasing function.[8]

[8] If both thresholds were positive, both (5) and (6) must hold. The left hand side of (6) is clearly higher than that of (5) while the right hand side of (6) is lower than that of (5) if the expected price function is

The final aspect of the behaviour of speculators that needs to be determined is the quantity that they buy or sell when aggregate output exceeds the relevant threshold, Q_t^L or $\widehat{Q}_t$. This quantity can be determined implicitly by the relationship between p_t and $p_t^e(H_{t+1})$ that must hold. We denote speculative sales in this case as $\widetilde{X}_t(Q_t)$, which is the solution in X to

$$a_t - X - Q_t = \delta \left(p_t^e(H_t - X) - w \right). \tag{7}$$

Applying the Implicit Function Theorem to (7) shows that $\widetilde{X}_t(Q_t)$ is strictly decreasing as long as p_t^e is decreasing, which is verified in the following sections. The higher the aggregate output, the smaller the market clearing clearing price is and the larger speculative purchases are (in absolute value), inducing larger final period inventories. Speculative sales are then given by

$$X_t^*(Q_t) = \begin{cases} H_t & \text{if} \quad Q_t \leq Q_t^L \text{ and } Q_t^L > 0 \\ -Q_t & \text{if} \quad Q_t \leq \widehat{Q}_t \text{ and } \widehat{Q}_t > 0 \\ \widetilde{X}_t(Q_t) & \text{otherwise.} \end{cases} \tag{8}$$

As only one of Q_t^L and $\widehat{Q}_t$ can be positive, only one of the first two conditions in (8) is possible for a given (H_t, a_t). The quantities Q_t^L and $\widetilde{X}_t(Q_t)$ are consistent with what is found in the competitive storage literature. Compared to that literature however, the existence of $\widehat{Q}_t$ is novel, as in our setting the possibility that consumers do not purchase must be considered. Studying this possibility is particularly important as the incentives of oligopolists to produce when consumer demand is low enough are entirely driven by the future expected price paid by speculators and not the value of consumers' current demand.

Given the behaviour of speculators in (8), we can now state the inverse demand net of speculation that producers face:

$$P_t(Q_t) = \begin{cases} a_t - H_t - Q_t & \text{if} \quad Q_{T-1} \leq Q_t^L \text{ and } Q_t^L > 0 \\ \delta(p_t^e(H_{t+1} + Q_t) - w) & \text{if} \quad Q_{T-1} \leq \widehat{Q}_t \text{ and } \widehat{Q}_t > 0 \\ a_t - \widetilde{X}_t(Q_t) - Q_t & \text{otherwise.} \end{cases} \tag{9}$$

With the behaviour of speculators determined by (8) and (9), payoffs to producers can be specified entirely in terms of output. The marginal payoff to a firm in any period will be discontinuous at an output that results in aggregate output of Q_t^L or $\widehat{Q}_t$, so the nature of the equilibrium in period t depends on where aggregate output falls in relation to these thresholds. First, a stockout may occur in which speculators sell their entire stocks and consumers buy the total of speculative and producer sales. We will denote this type of equilibrium with a

decreasing in future stocks, so both (5) and (6) cannot hold simultaneously.

C. Second, consumers may buy nothing and speculators purchase the entire output of firms, which we will denote with an S. Third, consumers may make some purchases and speculators carry inventory into the following period, denoted with a CS. Finally, there is the possibility that producers deter speculators by producing exactly Q_t^L in aggregate, which we will denote with an L.

It is not possible to find the equilibrium analytically for the general T-period game due to the difficulty in determining the expected price function.[9] In our game, the discontinuities in marginal profit also contribute to the difficulty in finding closed-form solutions. However, we are able to derive analytic results for the equilibrium in period $T-1$ which we turn to in the next section. Following that, we verify that the types of equilibria that we find in period $T-1$ exist also in earlier periods by solving the game with numerical approximation techniques in section 4.

3 Equilibrium in $T-1$

In this section we present results that characterize the equilibrium in period $T-1$ by deriving closed-form representations of the expected price function and equilibrium quantities. We start by presenting the equilibrium outcome for period T and then derive speculators' optimal behaviour and the implications for producers marginal profit. We then analyze the equilibrium in period $T-1$

3.1 Period T equilibrium

Since there is no salvage value to inventories held at the end of T, speculators will sell the quantity they have stored as long as $p_T > 0$. In this case aggregate speculative sales are equal to $X_T = H_T$. If on the other hand the aggregate quantity stored by speculators is such that the market price is equal to zero once H_T is sold, then the aggregate speculative sales cannot exceed the maximal quantity consumers are ready to buy given the aggregate production of firms, $a_T - Q_T$. In that case, aggregate speculative sales are simply $X_T = a_T - Q_T$ and the price is zero. Consequently, the inverse demand faced by producers in the final period is

$$P_T(Q_T) = \max\{a_T - H_T - Q_T, 0\}. \tag{10}$$

Given the inverse demand net of speculative sales, $P_T(Q_T)$, each producer chooses the quantity q_T^i to maximize its second period profit, $\pi_T^i = P_T(Q_T)q_T^i - \frac{c}{2}{q_T^i}^2$, with respect to the quantity produced q_T^i. The unique and symmetric pure strategy Nash equilibrium for the

[9] This is true even for the perfectly competitive case as has been known since Gustafson (1958)

final period subgame is given by the following functions of the period T state, (H_T, a_T):

$$q^*_T(H_T, a_T) = \frac{1}{n}(1-\beta)\max[a_T - H_T, 0], \quad Q^*_T(H_T, a_T) = (1-\beta)\max[a_T - H_T, 0], \tag{11}$$

$$p^*_T(H_T, a_T) = \beta \max[a_T - H_T, 0], \tag{12}$$

$$X^*_T(H_T, a_T) = \min[H_T, a_T], \tag{13}$$

with the final period value given by equilibrium profit:

$$V_T(H_T, a_T) = \pi^*_T(H_T, a_T) = \gamma(\max[a_T - H_T, 0])^2. \tag{14}$$

where $\beta = (1+c)/(n+1+c)$ and $\gamma = (2+c)/2(n+1+c)^2$ both belonging to $(0,1)$. For a given c, β reflects the impact of competition on the market price and γ reflects the impact of competition on each firm's profit in period T. The larger the number of producers the closer β and γ are to zero. The expected price function $p^e_{T-1}(H_T)$ can now be defined:[10]

$$\begin{aligned} p^e(H_T) \equiv E_{T-1}[P_T|H_T] &= \int_{H_T}^{A} \beta(a - H_T)/A \, da \\ &= (\beta A/2)(1 - H_T/A)^2 \end{aligned} \tag{15}$$

3.2 Implications for producer marginal profit

Before determining the equilibrium in period $T-1$ we need to understand the implications of speculators' behaviour, as given in (8) and (9), on producers' marginal profit. It is possible to do this analytically since we have closed-form solutions for Q^L, $\widehat{Q}$, and $\widetilde{X}(Q_{T-1})$.[11] Let $\Pi^i(q^i_{T-1}, Q^{-i}_{T-1})$ denote the total profit to firm i over the last two periods when it produces q^i_{T-1} and the other firms produce Q^{-i}_{T-1} in aggregate while incorporating the period T equilibrium:

$$\Pi^i(q^i_{T-1}, Q^{-i}_{T-1}) = P_{T-1}(Q_{T-1})q^i_{T-1} - \frac{c}{2}{q^i_{T-1}}^2 + \delta E_{T-1}[V_T(H_{T-1} - X^*_{T-1}(Q_{T-1}), a_T)] \tag{16}$$

with $Q_{T-1} = q^i_{T-1} + Q^{-i}_{T-1}$. As speculators' position, $X^*_{T-1}(Q_{T-1})$, and inverse demand, $P_{T-1}(Q_{T-1})$, are continuous[12] and piece-wise differentiable, individual profit is also continuous and piece-wise differentiable. The marginal profit of a firm choosing a level of output q^i_{T-1}, where it exists, is equal to

$$\Pi^i_q(q^i_{T-1}, Q^{-i}_{T-1}) = \frac{\partial P_{T-1}}{\partial Q_{T-1}} q^i_{T-1} + P_{T-1}(Q_{T-1}) - c\, q^i_{T-1} + \frac{2\delta\gamma}{\beta} \frac{dX^*_{T-1}}{dQ_{T-1}} p^e(H_{T-1} - X^*_{T-1}(Q_{T-1})) \tag{17}$$

[10]For the rest of this section, we drop the $T-1$ subscript on variables where there is no confusion.

[11]These are presented in Lemma 1 of Appendix B

[12]Continuity follows from the definitions of Q^L, $\widehat{Q}$, and $\widetilde{X}(Q_{T-1})$.

where the last term is from Lemma 2 of Appendix B. Marginal profit is discontinuous at the thresholds Q^L and $\widehat{Q}$ for two reasons: from (9), the marginal effect of output on price is discontinuous; and, from (8), the marginal effect on expected future profit is discontinuous through the effects on future inventories. When a stockout occurs, $0 \leq Q_{T-1} \leq Q^L$, no inventories are carried into the final period so expected period T profit is not affected by output and period $T-1$ marginal profit is simply that of a Cournot model with consumer demand reduced by H_{T-1}. We denote marginal profit in this case by $\Pi_q^C(q_{T-1}^i, Q_{T-1}^{-i})$. When inventories are carried into the final period, both price and future inventories are affected by a firm's choice of output in a manner that depends on whether consumers purchase the good or not. In the case that consumers are excluded from purchasing in $T-1$, $0 \leq Q_{T-1} \leq \widehat{Q}$, we denote marginal profit as $\Pi_q^S(q_{T-1}^i, Q_{T-1}^{-i})$, while when consumers purchase and inventories are carried into the final period we denote marginal profit as $\Pi_q^{CS}(q_{T-1}^i, Q_{T-1}^{-i})$.[13]

In order to find values of (H_{T-1}, a_{T-1}) for which a unique symmetric equilibrium exists, we work under an assumption that guarantees that marginal profit is strictly decreasing apart from the points at which aggregate output meets the thresholds Q_{T-1}^L and $\widehat{Q}_{T-1}$. Whereas this property is trivially satisfied when a stock-out occurs, it is not automatically the case when speculators carry inventories to the final period. A sufficient condition for marginal profit to be strictly decreasing in the S and CS cases is that production cost be sufficiently convex:

Assumption 1 (Cost convexity)**.** $c \geq \delta\beta$

This assumption ensures that when there is an equilibrium candidate of the S or CS type, it is the unique one of that type. The only potential problems for the existence or uniqueness of equilibrium are then associated with the discontinuities in marginal payoffs at Q_{T-1}^L and $\widehat{Q}_{T-1}$. Note that, since β is declining in n, Assumption 1 is more constraining for more concentrated market structures. This implies that, if Assumption 1 holds for $n = 1$, it will hold for all n. We can now summarize the characteristics of period $T-1$ marginal profit in $T-1$ with the following proposition:

Proposition 1. *The marginal profit of producer i, $\Pi_q^i(q_{T-1}^i, Q_{T-1}^{-i})$,*

(i) jumps up at the output level such that consumers start to buy the product, $q_{T-1}^i = \widehat{Q} - Q_{T-1}^{-i}$, whenever this output level exists and is positive;

(ii) jumps up or down at the output level such that speculators start to buy the product, $q_{T-1}^i = Q^L - Q_{T-1}^{-i}$, whenever this output level exists and is positive.

Furthermore, under Assumption 1, marginal profit for firm i is strictly decreasing when inventories are carried into the final period.

[13]The precise expressions for Π_q^C, Π_q^S, and Π_q^{CS} are given by (67), (65), and (66) in Appendix B.

Proof: See Appendix A.

Proposition 1 has important consequences for the nature of equilibrium. First, as is well known, the possibility of upward jumping marginal profit may generate multiple local solutions to the firm's optimization problem. Second, the possibility of an downward jump at $q^i_{T-1} = Q^L - Q^{-i}_{T-1}$ suggests that an equilibrium may exist in which all producers choose output levels such that the industry output is equal to Q^L. In this case a stockout is induced and the first period price is equal to $\delta(\beta A/2 - w)$. As Mitraille and Thille (2009) showed in this context for a monopoly, this limit outcome, L, is a possibility in an oligopoly as well.

In addition, Proposition 1 guarantees that a local solution to the first order condition is unique in the interior of the regions in which C, S, and CS are possible outcomes. To establish the existence of an equilibrium, we then need only check that no profitable deviation to another type of equilibrium (across the Q^L or $\widehat{Q}$ thresholds) exists.

3.3 Equilibrium

The equilibrium quantities for each of the different types of equilibria are now determined, beginning with the one in which consumers are excluded from the market. The consumer exclusion equilibrium, S, occurs when speculators' marginal valuation of the $T-1$ output is higher than a_{T-1}. We establish the existence of an S equilibrium in the following proposition:

Proposition 2. *For a_{T-1}, H_{T-1}, and n sufficiently small, an S equilibrium exists in which output for an individual firm is given by*

$$q_S = \frac{\delta\left(\beta - 2\gamma + \frac{\beta}{n}\right)(A - H_{T-1}) + \frac{c}{n}A - \sqrt{\Delta}}{\delta\left(\beta - 2\gamma + 2\frac{\beta}{n}\right)n}$$

with

$$\Delta = \left(-\frac{\delta\beta}{n}(A - H_{T-1}) + \frac{c}{n}A\right)^2 + 4\delta\left(\frac{\beta - 2\gamma}{2} + \frac{\beta}{n}\right)A\left(\delta w + \frac{c}{n}(A - H_{T-1})\right)$$

Proof: See Appendix A.

The S equilibrium occurs in circumstances where current demand, a_{T-1}, and existing inventories, H_{T-1}, are low compared to expected future demand, and only occurs for relatively concentrated market structures, which is due to the strategic incentive to increase $T-1$ sales by selling to speculators.

Turning now to the other types of equilibria, the existence of C, CS, and L has been demonstrated in the previous literature for monopolistic and perfectly competitive production. In the case of monopoly production, Mitraille and Thille (2009) show that all three of these are possible, while the large literature with competitive production shows that both C and CS

occur in that case. Not surprisingly, these equilibria may also occur in our oligopoly model and we present the conditions under which they obtain for period $T-1$ in Appendix B. Here we present the equilibrium output for each of these cases in period $T-1$.

The smoothing equilibrium candidate, CS, with individual firm output of q_{CS}, has both speculators carrying inventories to the final period and consumers buying the product in period $T-1$. It obtains when $\Pi_q^{CS}(q_{CS},(n-1)q_{CS})=0$ and we show in Appendix B that output is given by

$$q_{CS}=\frac{1}{n}\left(a_{T-1}-H_{T-1}+\delta w+A-A\left(-\frac{K_2}{3K_1}+Z^*\right)-\delta\beta\frac{A}{2}\left(-\frac{K_2}{3K_1}+Z^*\right)^2\right) \tag{18}$$

where Z^*, K_1, and K_2 are functions of a_{T-1}, H_{T-1} and model parameters.

The output levels for the equilibria in which speculators do not carry stocks into the final period are much simpler to obtain. In the stockout equilibrium, C, speculators sell all inventories H_{T-1} they own at the beginning of period $T-1$. Equilibrium quantities are defined by $\Pi_q^C(q_C,(n-1)q_C)=0$, which is simply the Cournot outcome when consumer demand is reduced by speculative inventories H_{T-1}. Individual producer output is then

$$q_C=\frac{1}{n}(1-\beta)\max[a_{T-1}-H_{T-1},0]. \tag{19}$$

Finally, speculation may be deterred. As Proposition 1 states, marginal profit may be jumping down at an aggregate output level equal to Q_{T-1}^L. It is possible that firms choose output levels lower than q_{CS}, ensuring that speculators sell all inventories in period $T-1$. When speculation deterrence occurs, individual firm production is

$$q_L=\frac{1}{n}\left(a_{T-1}-H_{T-1}-\delta\beta\frac{A}{2}+\delta w\right), \tag{20}$$

and in this case the equilibrium price is $p_{t-1}=\delta\beta A/2-\delta w$, which is the minimum price consistent with zero inventories being carried to the final period.

3.4 The non-monotonic effects of market structure

We now demonstrate that the incentive to sell to speculators is non-monotonic in the number of firms by examining two specific cases: i) the incentive to sell to speculators when they are the only source of demand, and ii) the existence of the L equilibrium. To examine the incentive to sell to speculators in isolation, it is useful to examine the S equilibrium described in Proposition 2 with $H_{T-1}=a_{T-1}=0$[14] and zero cost of production, $c=0$, in order to examine the marginal profit of an individual firm when aggregate production is zero. This

[14] The argument that follows does not rely on H_{T-1} and a_{T-1} being zero, just that they are low enough that Proposition 2 applies.

allows us to emphasize how the incentive to sell to speculators varies with the number of firms while abstracting from any other incentive to produce.[15] In this case, demand in $T-1$ can only come from speculators, who buy any output produced as long as p_{T-1} is not too high. To characterize the incentive for producers to sell to speculators in this setting, consider the marginal profit faced by a firm when industry output is in fact zero. In this case Π_q^S takes the relatively simple form:[16]

$$\Pi_q^S(0,0) = \delta\left((\beta - 2\gamma)\ A/2 - w\right) \tag{21}$$

Note that for $c = 0$, $\beta - 2\gamma = (n-1)/(n+1)^2$, which is zero for a monopolist: even though speculators would be willing to buy the commodity at a positive price, the monopolist will not supply them with output. This is not the case for an oligopoly. For $n > 1$, $\Pi_q^S(0,0)$ is strictly positive if the cost of storage is low enough and firms will want to produce positive output even with zero consumer demand. In addition, this effect is non-monotonic in n: $(n-1)/(n+1)^2$ reaches a maximum for $n = 3$ and stays above $1/9$ (its value when $n = 2$) for n between 2 and 5. It decreases to 0 as n goes to infinity. In this simple setting, this non-monotonicity is due to the strategic effect of competitive storage: by selling to speculators in $T-1$, firms capture some of the period T expected demand. This effect is absent under monopoly, where there is no competition for future demand, and under perfect competition, where the strategic effect is absent. To summarize this result we have

Proposition 3. *For a_{T-1} and H_{T-1} close to zero and $c = 0$, a firm's marginal profit when industry output is zero, $\Pi_q^S(0,0)$, is larger under oligopoly than under either monopoly or perfectly competitive market structures.*

For $c > 0$ this strategic motive to sell to speculators is balanced by the incentive to smooth production costs and the net effect of market structure on the equilibrium outcome depends on the relative strength of these two incentives.

We can also see the non-monotonic effect of the number of firms by examining the conditions under which the limit equilibrium, L, occurs. When $Q^L > 0$, the L equilibrium will exist if there is a feasible $a_{T-1} - H_{T-1}$ for which marginal profit is downward-jumping at $Q^L - (n-1)q_L$: $\Pi_q^C(q_{T-1}^L, (n-1)q_{T-1}^L) \geq 0$ and $\Pi_q^{CS}(q_{T-1}^L, (n-1)q_{T-1}^L) \leq 0$. From the expressions for Π_q^C and Π_q^{CS} given in Appendix B, these two conditions are satisfied simultaneously if

$$w \geq \frac{A(1+c)(n-1)}{2(n+1+c)^2}. \tag{22}$$

Clearly, for monopoly the right hand side of (22) is zero and there is a set of values for

[15] Clearly c=0 violates Assumption 1, but we are not attempting to compute the equilibrium in this particular case, rather we only want to isolate the incentive to sell to speculators.

[16] See (65) in Appendix B for the general form for Π_q^S.

$a_{T-1} - H_{T-1}$ in which an L equilibrium exists for any $w > 0$. Furthermore, the right hand side of (22) is non-monotonic in n, increasing at $n = 1$ and then decreasing to zero for large values of n, which means that it is less likely to be satisfied for market structures consisting of an intermediate number of firms. As the limit equilibrium represents the willingness of firms to deter speculation, we can summarize this result as:

Proposition 4. *The possibility that an equilibrium exists in which speculative storage is deterred is non-monotonic in n.*

With these two simple cases, we have shown that the strategic incentive to sell to speculators is non-monotonic in the number of firms. The general relationship between market structure and speculation in period $T-1$ equilibrium also depends on the desire to smooth production costs, so the net effect is difficult to discern in general. We now turn to a computed example to illustrate this relationship for a particular set of parameter values.

3.5 Example

Although we have explicit solutions for output in each type of equilibrium, the profit comparison required to rule out deviations to another equilibrium type are too complex to perform analytically. However, it is straightforward to compute them for a particular example, using the closed-form expressions for output determined above. We set $A = 20$, $\delta = 0.95$, $w = 0.2$, and choose $c = 0.6$ which satisfies Assumption 1 for $n \geq 1$. Consequently, we are certain that marginal profit for firm i is decreasing apart from the discontinuities at $Q^L - Q_{T-1}^{-i}$ or $\widehat{Q} - Q_{T-1}^{-i}$. We find the $T-1$ equilibrium as described above for $n = 1, 2, ..., 80$ and for 500 equally spaced values for $a_{T-1} \in [0, 20]$. The proportion of occurrences of each type of equilibrium for each value of n is illustrated in Figure 1 for zero initial stocks,[17] $H_{T-1} = 0$.

The C equilibrium is the the unique equilibrium for large first period demand and for large n, with more than 78 firms speculation is blockaded and only the C equilibrium is possible. The CS equilibrium occurs for a large set of parameters and is the unique equilibrium over a substantial range. There are a large number of situations for which we have multiple equilibria of either C and CS. An equilibrium of the CS type exists more than 50% of the time for a wide range of n, which implies that speculators are carrying inventory even though demand is expected to fall.

The consumer exclusion equilibrium (S) occurs over a significant range of states when the market is not very competitive. When it occurs, it is usually the unique outcome, although there are a relatively small number of cases in which either S or CS can be the equilibrium. The limit equilibrium (L) occurs for a small number of demand states when there is one firm or when there are 76–78 firms.

[17] We are most interested in what happens with zero initial stocks as this case allows a direct comparison to the Cournot equilibrium in the absence of storage.

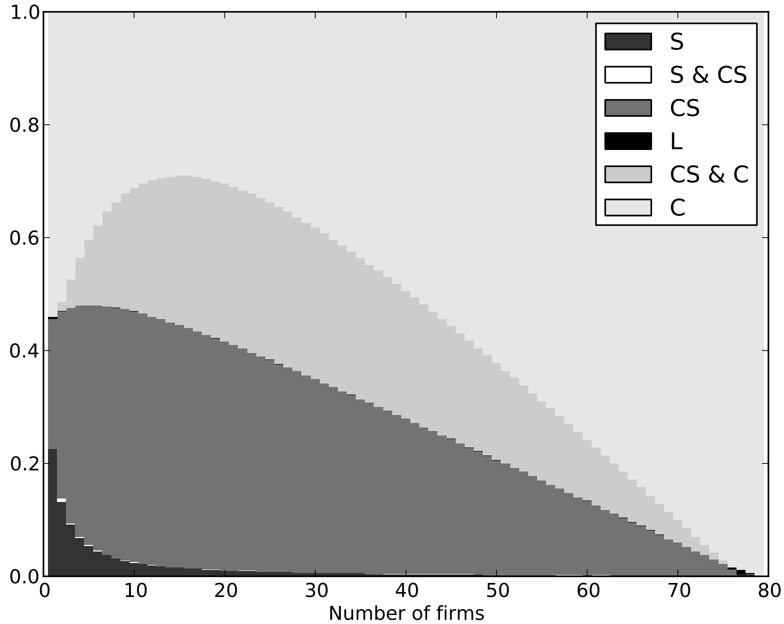

Figure 1: Equilibria in period $T-1$.

The proportion of demand states for which the CS equilibrium obtains in Figure 1 is non-monotonic in the number of firms. This is true even when we consider only the situations in which CS is the unique equilibrium. To examine the implications of non-monotonicity in the type of equilibrium for expected price and welfare, we compute speculative purchases, expected price and welfare for $n = 1, ..., 80$ and plot the results in Figure 2. The effect on prices and welfare are computed as the difference between the equilibrium values with speculation and those without, expressed as a proportion of the latter. We see that the level of speculative purchases is non-monotonic in the number of firms, peaking at around 10 firms and slowly declining to zero at over 70 firms. Expected period T price mirrors the level of stocks carried and clearly speculation has the effect of reducing period T prices. However, the effect on $T-1$ price is dependent on market structure: with relatively few firms speculation causes an increase in expected $T-1$ price, due largely to the relatively high prices when the equilibrium is S. With more firms competing, however, the effect of speculation is to reduce expected $T-1$ price as the S equilibrium occurs only rarely. The implications of these effects on welfare are shown in the lower right panel of Figure 2 in which we plot the effects of speculation on the discounted sum of expected welfare (consumer surplus plus producer profit) against the number of producers. Again we see a non-monotonic effect: welfare increases for relatively concentrated market structures, but actually falls for relatively unconcentrated ones. The magnitude of the welfare effects of speculation in this example is not large (at most under

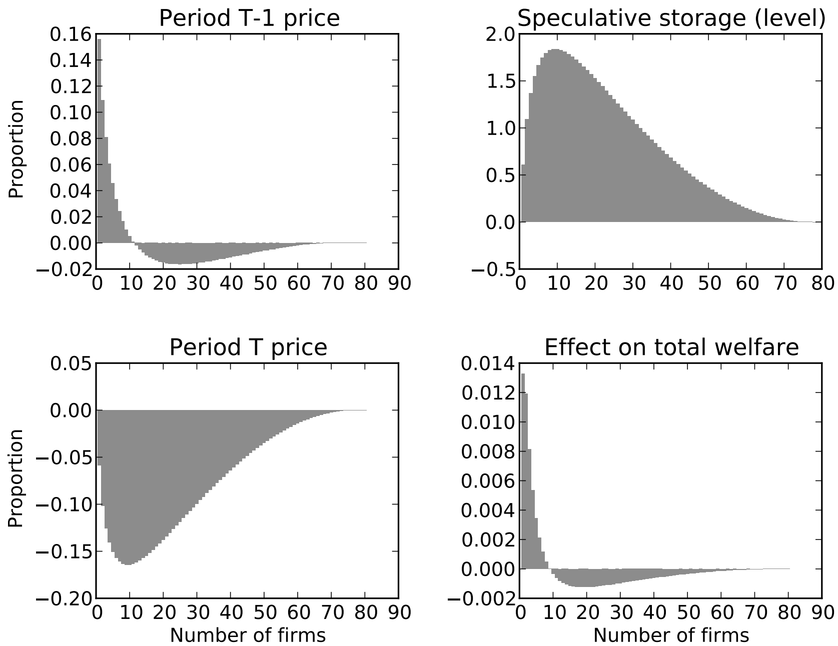

Figure 2: The effects of speculative storage on expected price and welfare relative to that in the absence of speculation.

2% of expected welfare in the absence of speculation) but there is potential for substantial redistribution between consumers and producers within periods as seen by the effects on the expected $T-1$ price, which can be as high as 15%.

To see the effects of higher storage costs and to demonstrate that the limit equilibrium need not be as rare as we see in Figure 1, in Figure 3 we plot the equilibria that obtain with a value of $w = 1.1$ and all other parameters the same as in Figure 1. Not surprisingly, equilibria with storage occur much less frequently and the stockout equilibrium is the unique outcome for $n > 12$. The L equilibrium occurs somewhat more frequently for the monopoly and when there are 10 to 12 firms. The higher cost of storage makes deterrence of speculators easier to attain while retaining the non-monotonicity discussed above.

4 Equilibrium for $t < T-1$

Explicit expressions for output in the $T-1$ equilibrium were possible due to the relatively simple equilibrium in period T. In particular, speculators' behaviour was quite simple in that they sell their stocks as long as $p_T > 0$, allowing us to compute the expectation of the period T price with little difficulty. In periods prior to $T-1$ however, we are not able to derive closed-form solutions for p_t^e and V_t^e due to more complex behaviour by speculators

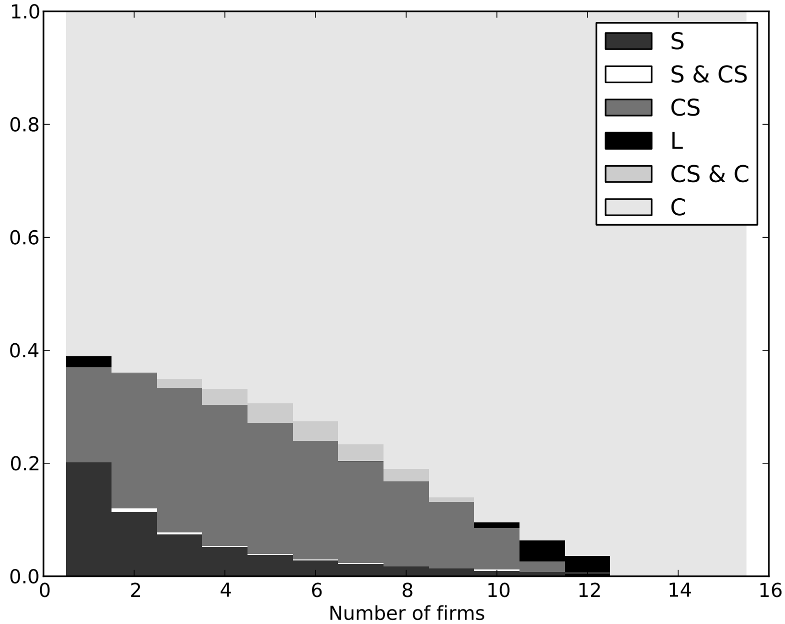

Figure 3: Equilibria in period $T-1$: Increased storage cost.

in the following period. Consequently, we turn to numerical approximation methods to find approximate equilibria to the game for periods $T-2$ and earlier in order to demonstrate the the analytic results established for period $T-1$ continue to obtain for $t < T-1$.

We apply the collocation method,[18] using cubic splines to approximate the expected price and value functions, denoting these approximations ρ_t and $\nu_{i,t}$. We start with the period T solution presented above and iterate backwards. Given a vector of m values for H_{t+1} between 0 and H_{max}, $\bar{H} = (0, H^1, ..., H_{max})$ we compute the vectors $\bar{p}^e_t$ and $\bar{V}^e_t$ which are the expected price and value[19] associated with future stocks given by each element of $\bar{H}$, i.e., for $k = 1, 2, ..., m$,

$$\bar{p}^e_{tk} = \int_0^A p^*_{t+1}(a, \bar{H}_k)/A da \tag{23}$$

and

$$\bar{V}^e_{tk} = \int_0^A V^*_{t+1}(a, \bar{H}_k)/A da \tag{24}$$

with $p^*_{t+1}(a, H)$ and $V^*_{t+1}(a, H)$ the equilibrium price and value functions in period $t+1$ given state (a, H) and period $t+1$ approximations ρ_{t+1} and ν_{t+1}. Our approximation to $p^e_t(H)$, $\rho_t(H)$, is found by fitting a cubic spline to the $\bar{H}$ and $\bar{p}^e_t$ vectors. Similarly, we fit a cubic

[18]Judd (1998), Ch. 11 provides a description of the method, which he applies to the competitive storage problem in Ch. 17.4

[19]We look for symmetric equilibria only, so the expected value function is identical for each firm.

spline to $\bar{H}$ and $\bar{V}_t^e$ to generate our approximation to V_t^e, $\nu_t(H)$.

In summary, to find the solution in any period t_0:

Step 0 Compute the period T equilibrium price and value, $p_T^*(a, \bar{H}_k)$ and $V_T^*(a, \bar{H}_k)$, $k = 1, ...m$, using (12) and (14). Set $t = T - 1$.

Step 1 For each $k = 1, ..., m$ compute

$$\bar{p}_{tk}^e = \int_0^A p_{t+1}^*(a, \bar{H}_k | \rho_{t+1}, \nu_{t+1})/A da, \tag{25}$$

$$\bar{V}_{tk}^e = \int_0^A V_{t+1}^*(a, \bar{H}_k | \rho_{t+1}, \nu_{t+1})/A da \tag{26}$$

Step 2 Fit a cubic spline to $(\bar{H}, \bar{p}_t^e)$ and $(\bar{H}, \bar{V}_t^e)$ to form $\rho_t(H)$ and $\nu_t(H)$

Step 3 While $t > t_0$, decrement t by one and return to Step 1.

When computing the equilibrium for period $t + 1$ an equilibrium selection is required for the cases in which multiple equilibria occur. We assume producers play C when both C and CS are possible and CS when both CS and S are possible.[20]

In order to analyze a representative period to check the robustness of the analytic results for the period $T - 1$ equilibrium, we choose a value for t_0 for which precedes T sufficiently for there to be little variation in the expected price function from period to period. In particular, we iterate until there is no more than a 0.1% variation in $\rho_t(\bar{H}_k)$ for each k between periods $t_0 + 1$ and t_0. The resulting t_0 depends on n, so we present results for the earliest t_0 over $n = 1, ..., 80$, which for the parameters used in our example gives $t_0 = T - 10$ when $n = 3$.

The equilibrium type that occurs in period $T - 10$ with $H_{T-10} = 0$ is illustrated in Figure 4. The qualitative nature of Figure 4 is similar to that of Figure 1, illustrating that the analytic results obtained for period $T - 1$ are robust to situations with a more distant time horizon. One notable difference between the two figures is that, while the set of circumstances in which the CS equilibrium obtains is similar, there is an increase in the number of states in which CS is unique relative to the multiple equilibria with C. Essentially, the set of states for which the stockout equilibrium, C, is possible has shrunk. The reason for this is that in $T - 1$ sales to speculators are guaranteed to be re-sold in the following period, while in $T - 10$ there is a possibility that they will not be re-sold, effectively lowering the cost to producers of allowing speculators to carry inventories. This effect reduces the incentives for firms to trigger a stockout. For similar reasons, the frequency of the consumer exclusion equilibrium, S, is larger than was the case in Figure 1: firms are more willing to allow speculative storage

[20] As with finitely repeated games, for which Benoit and Krishna (1985) show that a large set of subgame perfect equilibria can be sustained when there are multiple stage game equilibria, other subgame perfect equilibria may arise here due to the potential multiplicity of equilibria in any period prior to T.

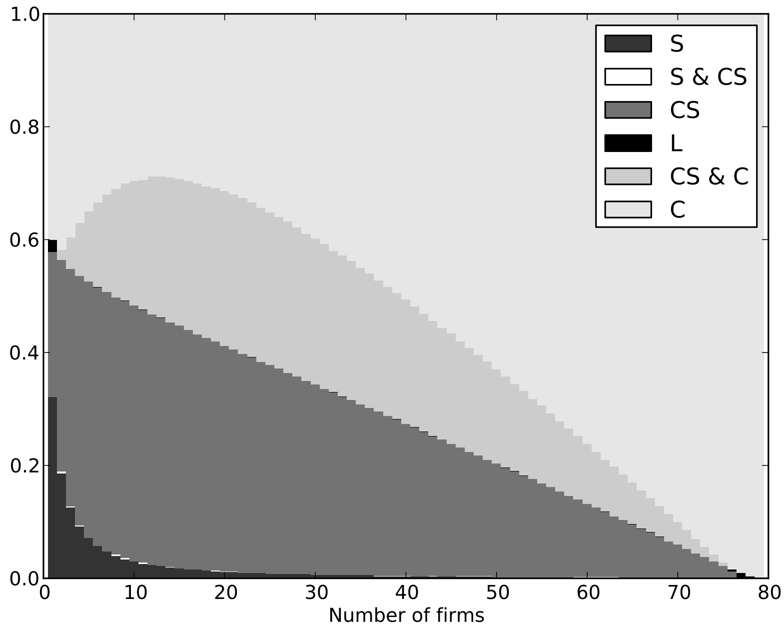

Figure 4: Equilibria in period $T - 10$.

resulting in a higher price which is more likely to cause consumers to withdraw from purchasing output.

5 Conclusion

Our results indicate that speculation storage is more likely to occur under an oligopoly market structure than under the extremes of monopoly and perfect competition. Furthermore, speculation can have a dramatic effect on the nature of the equilibrium in a non-cooperative setting. Of particular interest are the results that consumers may be excluded from purchasing in a particular period and that the degree of speculative storage can vary non-monotonically with market structure. We demonstrated with an example that this can cause non-monotonic effects on prices and welfare. Although the net effect on total welfare is modest, there are significant distributional effects of speculation between producers and consumers as well as over time and these effect vary with market structure. As many commodity markets feature oligopolistic production, our results suggest that the prevalence of concern about speculation may be warranted.

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Appendices

A Proofs

A.1 Proposition 1

Proof. (i) For $0 < q^i_{T-1} < \widehat{Q} - Q^{-i}_{T-1}$, $p_{T-1} > a_{T-1}$ and consumer demand is zero, while if $q^i_{T-1} > \widehat{Q} - Q^{-i}_{T-1}$, $p_{T-1} < a_{T-1}$ and both consumer and speculative demand is positive. Consequently, the individual firm's first period inverse demand flattens as the threshold $\widehat{Q} - Q^{-i}_{T-1}$ is crossed, which results in the firm's profit being less sensitive to increases in output for q^i_{T-1} larger than $\widehat{Q} - Q^{-i}_{T-1}$ than for smaller levels of output. Hence, marginal profit must take an upward jump at $q^i_{T-1} = \widehat{Q} - Q^{-i}_{T-1}$.

(ii) In this case the discontinuity in marginal profit occurs at $q^i_{T-1} = Q^L - Q^{-i}_{T-1}$. A stock-out occurs and only consumers buy the product when the output of firm i is lower than this value, and consequently the marginal profit at the left of the discontinuity is

$$\Pi^C_q(Q^L - Q^{-i}_{T-1}, Q^{-i}_{T-1}) = a_{T-1} - H_{T-1} - (2+c)Q^L + (1+c)Q^{-i}_{T-1}. \tag{27}$$

Since $H_T = 0$ is also equal to 0 to the right of $q^i_{T-1} = Q^L - Q^{-i}_{T-1}$, the marginal profit at the right of the discontinuity is

$$\Pi^{CS}_q(Q^L - Q^{-i}_{T-1}, Q^{-i}_{T-1}) = a_{T-1} - H_{T-1} - Q^L - \left(1 + c - \frac{1}{1+\delta\beta}\right)(Q^L - Q^{-i}_{T-1}) - \frac{2\gamma(a_{T-1} - H_{T-1} - Q^L + \delta w)}{\beta(1+\delta\beta)}. \tag{28}$$

Simplifying the last term with $a_{T-1} - H_{T-1} - Q^L + \delta w = \delta\beta A/2$ from the definition of Q^L, the difference across the discontinuity is equal to

$$\Pi^{CS}_q(Q^L - Q^{-i}_{T-1}, Q^{-i}_{T-1}) - \Pi^C_q(Q^L - Q^{-i}_{T-1}, Q^{-i}_{T-1}) = \frac{Q^L - Q^{-i}_{T-1} - \delta\gamma A}{1+\delta\beta}. \tag{29}$$

Clearly $Q^L - Q^{-i}_{T-1} > 0$ for the discontinuity to be relevant, but this difference may be larger or smaller than $\delta\gamma A$ and consequently the marginal profit may upward or downward jumping at $Q^L - Q^i_{T-1}$.

For the second part of Proposition 1, differentiating (65) yields

$$\Pi^S_{qq}(q^i_{T-1}, Q^{-i}_{T-1}) = \delta\left(1 - 2\frac{\gamma}{\beta}\right)\frac{dp^e(H_T)}{dH_T} - \delta\beta\left(1 - \frac{H_T}{A}\right) + \delta\beta\frac{q^i_{T-1}}{A} - cq^i_{T-1}, \tag{30}$$

where we have used $H_T = H_{T-1} + Q_{T-1}$ in S. From (15) we have

$$\frac{dp^e(H_T)}{dH_T} = -\beta\left(1 - \frac{H_T}{A}\right), \tag{31}$$

so we have

$$\Pi^S_{qq} = -2\delta(\beta-\gamma)\left(1 - \frac{H_T}{A}\right) + \delta\beta\frac{q^i_{T-1}}{A} - c. \tag{32}$$

The first term in (32) is negative since $\beta > \gamma$. By Assumption 1, $\delta\beta < c$, so $\delta\beta\frac{q^i_{T-1}}{A} < c$ since $q^i_{T-1} < A$ must be true. Consequently, $\Pi^S_{qq} < 0$.

The same analysis can be performed for $\Pi^{CS}_q(q^i_{T-1}, Q^{-i}_{T-1})$. Differentiating (66) with respect to q^i_{T-1} yields

$$\begin{aligned}\Pi^{CS}_{qq} = -\left(1 - 2\frac{\gamma}{\beta}\frac{d\widetilde{X}}{dQ_{T-1}}\right)\delta\frac{dp^e(H_T)}{dH_T}\frac{d\widetilde{X}}{dQ_{T-1}} - \left(1 + \frac{d\widetilde{X}}{dQ_{T-1}}\right)& \\ + 2\frac{\gamma}{\beta}\delta p^e(H_{T-1} - \widetilde{X}(Q_{T-1}))\frac{d^2\widetilde{X}}{dQ^2_{T-1}} - q_{T-1}\frac{d^2\widetilde{X}}{dQ^2_{T-1}} - c.&\end{aligned} \tag{33}$$

Considering the first term in (33), $\frac{dp^e(H_T)}{dH_T} < 0$ as established above, $\frac{d\widetilde{X}}{dQ_{T-1}} < 0$ from Lemma 1, and $\gamma < \beta$, so the first term is negative. From Lemma 1, $\frac{d\widetilde{X}}{dQ_{T-1}}$ is smaller than one in absolute value, so the second term in (33) is also negative. Also from Lemma 1, we have

$$\frac{d^2\widetilde{X}}{dQ^2_{T-1}} = -\frac{\delta\beta/A}{(1 + \delta\beta(1 - (H_{T-1} - \widetilde{X}(Q_{T-1}))/A)^3} \le 0, \tag{34}$$

so the third term in (33) is negative. Finally, under Assumption 1, we have

$$\delta\beta\frac{q^i_{T-1}/A}{(1 + \delta\beta(1 - (H_{T-1} - \widetilde{X}(Q_{T-1}))/A)^3} \quad < \delta\beta \le c. \tag{35}$$

since $q^i_{T-1} < A$. Consequently, $-q^i_{T-1}\frac{d^2\widetilde{X}}{dQ^2_{T-1}} - c \le 0$ and $\Pi^{CS}_{qq} < 0$. □

A.2 Proposition 2

Proof. To prove existence, we note that a symmetric S equilibrium exists if the following conditions are satisfied:

I. $\widehat{Q} > 0$.

II. A solution, q^S, satisfying $\Pi^S_q(q^S, (n-1)q^S) = 0$ exists with $q^S \in (0, \widehat{Q}/n)$. By Proposition 1, a firm's marginal profit is decreasing in its own output, which results in unique and continuous best responses. Consequently, $q^S \in (0, \widehat{Q}/n)$ if

(a) a producer's best response to its competitors producing zero output is to produce positive output. This is ensured if

$$\Pi_q^S(0,0) > 0, \tag{36}$$

(b) a producer's best response to its competitors each producing $\widehat{Q}/n$ is to produce less than $\widehat{Q}/n$. This is ensured it

$$\Pi_q^S(\widehat{Q}/n, (n-1)\widehat{Q}/n) < 0. \tag{37}$$

III. Producers do not wish to deviate by increasing output to induce sales to consumers. Proposition 1 establishes that marginal profit jumps upward at $q = \widehat{Q} - (n-1)\widehat{Q}/n$, so there may a profitable deviation into the CS region. A sufficient (but not necessary) condition to rule such a deviation out is

$$\Pi_q^{CS}(\widehat{Q} - (n-1)q^S, (n-1)q^S) < 0. \tag{38}$$

We examine the conditions under which each of these conditions holds in turn.

First, condition I is satisfied if

$$a_{T-1} < \left(1 - \frac{H_{T-1}}{A}\right)^2 \frac{\delta\beta}{2} A - \delta w. \tag{39}$$

As the right hand side of this inequality is decreasing in both H_{T-1} and n (since β is decreasing in n), (39) holds for small but positive values of a_{T-1}, H_{T-1}, and n.

Condition IIa requires

$$(\beta - 2\gamma)\frac{A}{2}\left(1 - \frac{H_{T-1}}{A}\right)^2 - w > 0 \tag{40}$$

or

$$H_{T-1} < A\left(1 - \sqrt{\frac{2w}{A(\beta - 2\gamma)}}\right) \tag{41}$$

which is independent of a_{T-1}. As $\beta - 2\gamma$ is decreasing in n, the right hand side of (41) is decreasing in n, and so is more likely to hold for small H_{T-1} and small n.

Condition IIb requires

$$\delta(\beta - 2\gamma)\frac{A}{2}\left(1 - \frac{H_{T-1} + \widehat{Q}}{A}\right)^2 - \delta w - \left(\delta\beta\left(1 - \frac{H_{T-1} + \widehat{Q}}{A}\right) + c\right)\frac{\widehat{Q}}{n} < 0. \tag{42}$$

Since the final term on the left hand side of (42) is negative, it suffices to demonstrate that the first two terms sum to a negative quantity. From the definition of $\widehat{Q}$ in Lemma 1, we have $1 - \frac{H_{T-1}+\widehat{Q}}{A} = \sqrt{\frac{a_{T-1}+\delta w}{\delta\beta A/2}}$, so the first two terms of (42) will be negative if

$$(\beta - 2\gamma)\frac{a_{T-1} + \delta w}{\delta\beta} - w < 0, \tag{43}$$

or

$$a_{T-1} < \frac{2\delta\gamma}{\beta - 2\gamma}w. \tag{44}$$

Since the right hand side of (44) is strictly positive, there are values of $a_{T-1} > 0$ that satisfy (44). Consequently there are positive values of a_{T-1} for which (42) and hence (37) are satisfied. Note that since $\frac{2\gamma}{\beta-2\gamma} = \frac{2+c}{(1+c)(n+1+c)-(2+c)}$, the right hand side of (44) is decreasing in n, so the more concentrated the market is, the larger the range of a_{T-1} for which (44) holds.

Using (66) and (15), condition III requires

$$a_{T-1} < \left(1 + \frac{d\widetilde{X}_{T-1}}{dQ_{T-1}} + c\right)(\widehat{Q} - (n-1)q^S) + \frac{2\gamma(a_{T-1} + \delta w)}{\beta\left(1 + \delta\beta\left(1 - \frac{H_{T-1}+\widehat{Q}}{A}\right)\right)} \tag{45}$$

where we have used $\widetilde{X}(\widehat{Q}) = -\widehat{Q}$. Since $-1 < \frac{d\widetilde{X}_{T-1}}{dQ_{T-1}} < 0$ from Lemma 1 and $\widehat{Q} > (n-1)q^S$ in a feasible S equilibrium, the right hand side of (45) is strictly positive. Consequently, (45) holds for a sufficiently small, but positive value for a_{T-1}.

To demonstrate that all four conditions can be satisfied simultaneously, note that inequalities (39), (44), and (45) define a subset of a_{T-1} bounded between zero and a value which is positive for sufficiently small H_{T-1} and n. In addition, (41) holds for sufficiently small values of H_{T-1} and n. Hence, the set of values of a_{T-1}, H_{T-1} and n which satisfy conditions I – III is non-empty, and an S equilibrium exists in this game.

To determine the equilibrium level of output, we find q^S that solves $\Pi_q^S(q^S, (n-1)q^S) = 0$. From (65) and (15) this becomes

$$\delta(\beta - 2\gamma)\frac{A}{2}\left(1 - \frac{H_{T-1} + Q_{T-1}}{A}\right)^2 - \delta w - \delta\beta\left(1 - \frac{H_{T-1} + Q_{T-1}}{A}\right)q^S - cq^S = 0, \tag{46}$$

which, using $H_T^S = H_{T-1} + Q_{T-1}^S$, becomes

$$\delta(\beta - 2\gamma)\frac{A}{2}\left(1 - \frac{H_T^S}{A}\right)^2 - \delta w - \delta\beta\left(1 - \frac{H_T^S}{A}\right)\frac{H_T^S - H_{T-1}}{n} - c\frac{H_T^S - H_{T-1}}{n} = 0. \tag{47}$$

Making the substitution $H_T^S = A - A\left(1 - \frac{H_T^S}{A}\right)$ for the last two occurrences of H_T^S in (47) allows us to express the solution as a second degree polynomial in $Z_T \equiv 1 - \frac{H_T^S}{A}$, which is the probability that demand exceeds period T inventories:

$$\delta(\beta-2\gamma)\frac{A}{2}Z_T^2-\delta w-\delta\beta Z_T\frac{A(1-Z_T)-H_{T-1}}{n}-c\frac{A(1-Z_T)-H_{T-1}}{n}=0. \tag{48}$$

Collecting terms, this can be written as $K_1Z_T^2+K_2Z_T+K_3=0$ with

$$\begin{aligned} K_1&=\delta\left(\frac{\beta-2\gamma}{2}+\frac{\beta}{n}\right)A\\ K_2&=-\frac{\delta\beta}{n}(A-H_{T-1})+\frac{c}{n}A\\ K_3&=-\delta w-\frac{c}{n}(A-H_{T-1}) \end{aligned} \tag{49}$$

Note that K_1 is positive and K_3 is negative[21], so the discriminant, $K_2^2-4K_1K_3$, is positive. There are two solutions to consider, and as Z_T is a probability, we must keep the one between 0 and 1. The solutions are $Z_T'=(-K_2-\sqrt{(K_2)^2-4K_1K_3})/2K_1$ and $Z_T''=(-K_2+\sqrt{(K_2)^2-4K_1K_3})/2K_1$, where Z_T' is negative[22]. Therefore we have

$$Z_T''=1-\frac{H_T^S}{A}=\frac{\frac{\delta\beta}{n}(A-H_{T-1})-\frac{c}{n}A+\sqrt{\Delta}}{\delta(\beta-2\gamma+2\beta/n)A} \tag{50}$$

with

$$\Delta=\left(-\frac{\delta\beta}{n}(A-H_{T-1})+\frac{c}{n}A\right)^2+4\delta\left(\frac{\beta-2\gamma}{2}+\frac{\beta}{n}\right)A\left(\delta w+\frac{c}{n}(A-H_{T-1})\right) \tag{51}$$

The aggregate quantity produced is then equal to

$$\begin{aligned} Q_{T-1}^S=&H_T^S-H_{T-1}=A-A\left(1-\frac{H_T^S}{A}\right)-H_{T-1}\\ =&A-H_{T-1}-\frac{\frac{\delta\beta}{n}(A-H_{T-1})-\frac{c}{n}A+\sqrt{\Delta}}{\delta\left(\beta-2\gamma+2\frac{\beta}{n}\right)}\\ =&\frac{\delta\left(\beta-2\gamma+\frac{\beta}{n}\right)(A-H_{T-1})+\frac{c}{n}A-\sqrt{\Delta}}{\delta\left(\beta-2\gamma+2\frac{\beta}{n}\right)} \end{aligned} \tag{52}$$

and $q_{T-1}^S=Q_{T-1}^S/n$ completes the proof. □

[21]For K_3 to be positive, it is necessary that $A-H_{T-1}<0$, that is it is necessary that inventories H_{T-1} exceed the maximal demand A; but this would force the price to 0 in period T and make any increase in inventories in period $T-1$ not profitable anyway, a contradiction to the case we are examining.

[22]Indeed either $K_2>0$ and Z_T' is obviously negative, or $K_2\le 0$ but in that case as $K_3<0$ the square root of the discriminant exceeds $-K_2$, that is $Z_T'<0$.

B $T-1$ equilibrium details

In this appendix we provide detailed derivations of equilibrium quantities for period $T-1$. We first examine speculators behaviour and then present producers marginal profit and finally derive the equilibrium output for each type of equilibria.

B.1 Speculator behaviour

The quantities that characterize speculators' behaviour is summarized in the following lemma:

Lemma 1. *There exist unique* Q^L, $\widehat{Q}$, *and* $\widetilde{X}(Q_{T-1})$ *given by*

$$\begin{aligned} Q^L &= a_{T-1} - H_{T-1} - \delta\beta\frac{A}{2} + \delta w, \\ \widehat{Q} &= A\left(1 - \sqrt{\frac{a_{T-1}+\delta w}{\delta\beta A/2}}\right) - H_{T-1}, \textit{and} \\ \widetilde{X}_{T-1}(Q_{T-1}) &= H_{T-1} + \frac{\sqrt{(1+2\delta\beta)A^2 + 2(a_{T-1} - H_{T-1} + \delta w - Q_{T-1})\delta\beta A} - (1+\delta\beta)A}{\delta\beta} \end{aligned}$$

with the properties that

(i)

$$\frac{d\widetilde{X}}{dQ_{T-1}} = -\frac{1}{1+\delta\ \beta(1-(H_{T-1}-\widetilde{X}(Q_{T-1}))/A)} < 0$$

(ii) $\widehat{Q}$ *and* Q^L *cannot be both positive: either one is positive and the other negative, or both are negative.*

Proof. The expression (and uniqueness) of Q^L is straightforward from using (15) in (5):

$$Q^L = a_{T-1} - H_{T-1} - \delta(\beta A/2 - w). \tag{53}$$

For $\widehat{Q}$, using (15) in (6) we have

$$\delta\beta\frac{A}{2}\left(1 - \frac{H_{T-1}+\widehat{Q}_{T-1}}{A}\right)^2 - \delta w - a_{T-1} = 0 \tag{54}$$

giving immediately

$$\widehat{Q}_{T-1} = A\left(1 - \sqrt{\frac{a_{T-1}+\delta w}{\delta\beta A/2}}\right) - H_{T-1}. \tag{55}$$

To determine $\widetilde{X}_{T-1}(Q_{T-1})$, using (15) in (7) yields

$$a_{T-1} - X - Q_{T-1} + \delta w - \delta\beta\frac{A}{2}\left(1 - \frac{H_{T-1}-X}{A}\right)^2 = 0 \tag{56}$$

Adding the difference $H_{T-1} - H_{T-1}$ to the left-hand-side and using $H_T = H_{T-1} - X$ we can express this as

$$a_{T-1} - H_{T-1} - Q_{T-1} + \delta w + A - A\left(1 - \frac{H_T}{A}\right) - \delta\beta\frac{A}{2}\left(1 - \frac{H_T}{A}\right)^2 = 0 \tag{57}$$

which is equivalent, denoting $Z_T = 1 - \frac{H_T}{A}$, to solve in Z_T :

$$a_{T-1} - H_{T-1} - Q_{T-1} + \delta w + A - AZ_T - \delta\beta\frac{A}{2}Z_T^2 = 0. \tag{58}$$

The discriminant of (58) can be written as $\Delta = (1+2\delta\beta)A^2 + 2(a_{T-1} - H_{T-1} + \delta w - Q_{T-1})\delta\beta A$. If $Q_{T-1} < a_{T-1} - H_{T-1} + \delta w + \frac{1+2\delta\beta}{\delta\beta}\frac{A}{2}$, then $\Delta > 0$ and the only potentially positive root is

$$Z'_T = \frac{\sqrt{\Delta} - A}{\delta\beta A} = \frac{\sqrt{(1+2\delta\beta)A^2 + 2(a_{T-1} - H_{T-1} + \delta w - Q_{T-1})\delta\beta A} - A}{\delta\beta A}, \tag{59}$$

and speculators position X solves

$$1 - \frac{H_{T-1} - X}{A} = \frac{\sqrt{(1+2\delta\beta)A^2 + 2(a_{T-1} - H_{T-1} + \delta w - Q_{T-1})\delta\beta A} - A}{\delta\beta A}$$

or

$$\widetilde{X}_{T-1}(Q_{T-1}) = H_{T-1} + \frac{\sqrt{(1+2\delta\beta)A^2 + 2(a_{T-1} - H_{T-1} + \delta w - Q_{T-1})\delta\beta A} - (1+\delta\beta)A}{\delta\beta} \tag{60}$$

If $Q_{T-1} \geq a_{T-1} - H_{T-1} + \delta w + \frac{1+2\delta\beta}{\delta\beta}\frac{A}{2}$, then $\Delta < 0$ and no real roots exist for (58); moreover, the left hand side of (58) is negative for all Z_T, meaning that the first period price is lower than the expected second period price net of the cost of storage: speculators would prefer to buy an infinite amount in that case.

Property *(i)* results application of the Implicit Function Theorem to (7) or via direct differentiation of (60).

To prove property *(ii)*, we need to study the sign of each threshold. From (53),

$$Q^L \geq 0 \quad \Leftrightarrow \quad a_{T-1} \geq H_{T-1} + \delta\beta A/2 - \delta w, \tag{61}$$

and from (55),

$$\widehat{Q}_{T-1} \geq 0 \quad \Leftrightarrow \quad a_{T-1} \leq \left(1 - \frac{H_{T-1}}{A}\right)^2 \frac{\delta\beta}{2}A - \delta w. \tag{62}$$

For $H_{T-1} = 0$, the right hand sides of (61) and (62) are equal, so clearly only one of Q^L and $\widehat{Q}$ is be positive. For $H_{T-1} > 0$, note that the right hand side of (61) is increasing in H_{T-1} while that of (62) is decreasing in H_{T-1}. Consequently, there are only three possibilities: i)

$Q^L > 0$ and $\widehat{Q} < 0$, ii) $Q^L < 0$ and $\widehat{Q} > 0$, and iii) $Q^L < 0$ and $\widehat{Q} < 0$. □

B.2 Producer marginal profits

The effects of period $T-1$ output choice on expected period T profit is summarized in the following lemma:

Lemma 2. *When speculators carry stocks into period T, $H_T > 0$, a producer's expected period T profit is decreasing in aggregate $T-1$ production, Q_{T-1}:*

$$\frac{\partial E_{T-1}[\pi_T^*]}{\partial Q_{T-1}} = 2\frac{\gamma}{\beta}\frac{dX_{T-1}^*}{dQ_{T-1}} p^e(H_{T-1} - X_{T-1}^*(Q_{T-1})) \leq 0$$

where dX_{T-1}^*/dQ_{T-1} is equal to -1 when speculators buy the entire period $T-1$ production, or to $d\widetilde{X}/dQ_{T-1} \in (-1, 0)$ when both consumers and speculators buy in period $T-1$.

Proof. Using (14), expected second period profit is

$$E_{T-1}[\pi_T^*(H_T, a_T)] = \gamma \int_{H_T}^{A} \frac{(a - H_T)^2}{A} da = \gamma \frac{A^2}{3}\left(1 - \frac{H_T}{A}\right)^3 \tag{63}$$

As $H_T = H_{T-1} - X_{T-1}^*(Q_{T-1})$, we have

$$\frac{\partial E_{T-1}[\pi_T^*]}{\partial Q_{T-1}} = \gamma A \left(-\frac{\partial H_T}{\partial Q_{T-1}}\right)\left(1 - \frac{H_T}{A}\right)^2 = 2\gamma \frac{\partial X_{T-1}^*}{\partial Q_{T-1}} \frac{p^e(H_T)}{\beta} \tag{64}$$

where $A\left(1 - \frac{H_T}{A}\right)^2$ has been replaced by $2p^e(H_T)/\beta$ using (15). □

We now present the marginal profit of a producer's choice of output in $T-1$ depending on which regime of C, S, or CS obtains. When $a_{T-1} - H_{T-1} \leq \delta(\beta A/2 - w)$ speculators always carry stocks and consumers are excluded from the market when total output is lower than $\widehat{Q}$. The marginal profit of firm i for an output level $q_{T-1}^i \leq \widehat{Q} - Q_{T-1}^{-i}$ is equal to

$$\Pi_q^S(q_{T-1}^i, Q_{T-1}^{-i}) \equiv \delta\left(1 - 2\frac{\gamma}{\beta}\right) p^e(H_{T-1} + Q_{T-1}) - \delta w - \delta\beta\left(1 - \frac{H_{T-1} + Q_{T-1}}{A}\right) q_{T-1}^i - c q_{T-1}^i, \tag{65}$$

where we use an S superscript to denote case in which speculators purchase the entire output. If $q_{T-1}^i \geq \widehat{Q} - Q_{T-1}^{-i}$ marginal profit is equal to

$$\Pi_q^{CS}(q_{T-1}^i, Q_{T-1}^{-i}) \equiv \delta\left(1 + 2\frac{\gamma}{\beta}\frac{d\widetilde{X}}{dQ_{T-1}}\right) p^e(H_{T-1} - \widetilde{X}(Q_{T-1})) - \delta w - \left(1 + \frac{d\widetilde{X}}{dQ_{T-1}}\right) q_{T-1}^i - c q_{T-1}^i, \tag{66}$$

where we use a CS superscript to denote the case in which both consumers purchase and speculators carry stocks into the final period.

When $a_{T-1} - H_{T-1} \geq \delta(\beta A/2 - w)$ consumers always purchase positive quantities and a stockout occurs when total output is lower than Q^L. The marginal profit of firm i for an output level $q^i_{T-1} \leq Q^L - Q^{-i}_{T-1}$ is equal to

$$\Pi^C_q(q^i_{T-1}, Q^{-i}_{T-1}) \equiv a_{T-1} - H_{T-1} - 2q^i_{T-1} - Q^{-i}_{T-1} - cq^i_{T-1}, \tag{67}$$

where a C superscript denotes the case in which consumers purchase the entire $T-1$ output plus speculative stocks. Finally, if $q^i_{T-1} \geq Q^L - Q^{-i}_{T-1}$ then the marginal profit is again given by equation (66).

B.3 Derivation of q_{CS}

Output in a CS equilibrium obtains when $\Pi^{CS}_q(q_{CS}, (n-1)q_{CS}) = 0$ and is detailed in the following proposition:

Proposition 5. *In a smoothing equilibrium, a firm individual output is equal to*

$$q_{CS} = \frac{1}{n}\left(a_{T-1} - H_{T-1} + \delta w + A - A\left(-\frac{K_2}{3K_1} + Z^*\right) - \delta\beta\frac{A}{2}\left(-\frac{K_2}{3K_1} + Z^*\right)^2\right)$$

where Z^* is the solution[23] *to $Z^3 + EZ + F = 0$ for which $\frac{K_2}{3K_1} \leq Z^* \leq 1 + \frac{K_2}{3K_1}$, and where*

$$E = \frac{1}{K_1}\left(K_3 - \frac{K_2^2}{3K_1}\right),\ F = \frac{1}{K_1}\left(K_4 + 2\frac{K_2^3}{27K_1^2} - \frac{K_3K_2}{3K_1}\right),$$

$$\begin{aligned}
\text{with } K_1 &= \delta(1+c)\delta\beta\frac{A}{2},\\
K_2 &= (\delta(1+c) + \delta\beta(1+2c) - 2n\delta\gamma)\,\frac{A}{2},\\
K_3 &= -\delta(1+c)\delta w + (c - (1+c)\delta\beta)A - (1+c)\delta\beta(a_{T-1} - H_{T-1})\\
K_4 &= -(n+c)\delta w - cA - c(a_{T-1} - H_{T-1}).
\end{aligned}$$

Proof. In a symmetric CS equilibrium output is determined by the solution to the first order condition $\Pi^{CS}_q(q^{CS}_{T-1}, (n-1)q^{CS}_{T-1}) = 0$. Since $p_{T-1} = a_{T-1} - \widetilde{X}_{T-1}(Q_{T-1}) - Q_{T-1} = \delta(p^e(H_T) - w)$ in CS with $p^e(H_T)$ given by (15), this reduces to

$$\delta\beta\frac{A}{2}\left(1 - \frac{H_T}{A}\right)^2 - \delta w - \left(1 + \frac{d\widetilde{X}_{T-1}}{dQ_{T-1}}\right)q^i_{T-1} - cq^i_{T-1} - \frac{2\gamma\delta\beta\frac{A}{2}\left(1 - \frac{H_T}{A}\right)^2}{\beta\left(1 + \delta\beta\left(1 - \frac{H_T}{A}\right)\right)} = 0. \tag{68}$$

[23] The specific closed form solution to this cubic equation is presented in the proof for brevity.

Using $Z_T = 1 - H_T/A$, we use (58) express aggregate production as

$$Q_{T-1} = a_{T-1} - H_{T-1} + \delta w + A - AZ_T - \delta\beta\frac{A}{2}Z_T^2, \tag{69}$$

which can be used with Lemma 1 to get

$$\frac{d\widetilde{X}_{T-1}}{dQ_{T-1}} = \frac{-1}{1+\delta\beta(1-F(H_{T-1}-\widetilde{X}_{T-1}))} = \frac{-1}{1+\delta\beta Z_T}. \tag{70}$$

We can now express the zero marginal payoff condition as

$$\delta\beta\frac{A}{2}Z_T^2 - \delta w - \left(1+c-\frac{1}{1+\delta\beta Z_T}\right)q_{T-1}^{CS} - \frac{2\gamma\delta\beta\frac{A}{2}Z_T^2}{\beta\left(1+\delta\beta Z_T\right)} = 0. \tag{71}$$

Since $q_{T-1}^{CS} = Q_{T-1}^{CS}/n$ in a symmetric equilibrium, using (69) we have

$$\delta\beta\frac{A}{2}Z_T^2 - \delta w - \left(1+c-\frac{1}{1+\delta\beta Z_T}\right)\frac{a_{T-1}-H_{T-1}+\delta w+A-AZ_T-\delta\beta\frac{A}{2}Z_T^2}{n} - \frac{2\delta\gamma\frac{A}{2}Z_T^2}{1+\delta\beta Z_T} = 0, \tag{72}$$

which is a cubic equation in Z_T that we write as $K_1Z_T^3 + K_2Z_T^2 + K_3Z_T + K_4 = 0$, with

$$K_1 = (n+1+c)(\delta\beta)^2\frac{A}{2} = \delta(1+c)\delta\beta\frac{A}{2} \tag{73}$$

$$\begin{aligned} K_2 &= n\delta\beta\frac{A}{2} - 2n\delta\gamma\frac{A}{2} + \delta\beta c\frac{A}{2} + A(1+c)\delta\beta \\ &= \left(\delta\beta(n+c+2(1+c)) - 2n\delta\gamma\right)\frac{A}{2} \\ &= \left(\delta(1+c) + \delta\beta(1+2c) - 2n\delta\gamma\right)\frac{A}{2} \end{aligned} \tag{74}$$

$$\begin{aligned} K_3 &= -n\delta w\delta\beta + cA - (1+c)\delta\beta(a_{T-1} - H_{T-1} + \delta w + A) \\ &= -\delta(1+c)\delta w + (c-(1+c)\delta\beta)A - (1+c)\delta\beta(a_{T-1} - H_{T-1}) \end{aligned} \tag{75}$$

$$\begin{aligned} K_4 &= -n\delta w - c(a_{T-1} - H_{T-1} + \delta w + A) \\ &= -(n+c)\delta w - cA - c(a_{T-1} - H_{T-1}) \end{aligned} \tag{76}$$

To determine the solution, we can first change the variable $Z_T = Y - \frac{K_2}{3K_1}$. This first change leads to the depressed cubic equation

$$Y^3 + \frac{1}{K_1}\left(K_3 - \frac{K_2^2}{3K_1}\right)Y + \frac{1}{K_1}\left(K_4 + 2\frac{K_2^3}{27K_1^2} - \frac{K_3K_2}{3K_1}\right) = 0 \tag{77}$$

Then, defining

$$E = \frac{1}{K_1}\left(K_3 - \frac{(K_2)^2}{3K_1}\right) \tag{78}$$

$$F = \frac{1}{K_1}\left(K_4 + 2\frac{(K_2)^3}{27(K_1)^2} - \frac{K_3K_2}{3K_1}\right), \tag{79}$$

the cubic equation we need to solve is:

$$Y^3 + EY + F = 0. \tag{80}$$

The following defining states the solution to a depressed cubic equation

Definition 1. *[CRC (1996)] Let $\mathcal{P}(Z) = Z^3 + E \cdot Z + F$ a third degree polynomial. The three solutions $\{Z_k\}_{k=0,1,2}$ to $\mathcal{P}(Z) = 0$ are:*

1. *if $27F^2 + 4E^3 \geq 0$, given by the real quantity:*

$$Z_0 = \sqrt[3]{-\frac{F}{2} + \sqrt{\frac{F^2}{4} + \frac{E^3}{27}}} + \sqrt[3]{-\frac{F}{2} - \sqrt{\frac{F^2}{4} + \frac{E^3}{27}}},$$

   and the two other roots are complex,

2. *if $27F^2 + 4E^3 < 0$, given by the quantities:*

$$Z_k = 2\sqrt{-\frac{E}{3}}\cos\left(\frac{1}{3}\arccos\left(\frac{3F}{2E}\sqrt{-\frac{3}{E}} - k\frac{2\pi}{3}\right)\right) \text{ for } k = 0, 1, 2,$$

   which are all real.

From Definition 1, depending on the sign of $27E^3 + 4F^2$, there can be up to three real solutions to this equation. As we are searching for a probability, we select the solution Y^* which verifies the fact that $Z_T = Y^* - \frac{K_2}{3K_1}$ belongs to $[0, 1]$, that is $\frac{K_2}{3K_1} \leq Y^* \leq 1 + \frac{K_2}{3K_1}$. □

B.4 Characterization of $T - 1$ equilibrium

In this appendix we determine the conditions governing the existence of the alternative types of equilibria in terms of values of (H_{T-1}, a_{T-1}). As a complete characterization is not possible analytically due to the difficulty in comparing deviation profits when marginal profit is upward-jumping at Q^L or $\widehat{Q}$, in this situation we demonstrate how the optimal deviation output is calculated when computing the results for the example in the text. There are three broad situations to examine depending on the signs of Q^L and $\widehat{Q}$ which we consider in turn.

B.4.1 Q^L, $\widehat{Q} < 0$

The case in which both Q^L and $\widehat{Q}$ are negative corresponds to a situation in which q_{CS} is the unique symmetric Nash equilibrium to the game. This happens if:

$$\left(1 - \frac{H_{T-1}}{A}\right)^2 \frac{\delta\beta}{2} A - \delta w \leq a_{T-1} \leq H_{T-1} + \delta\beta A/2 - \delta w \tag{81}$$

As the marginal profit is continuous and decreasing in the individual quantity, there are no global deviations to consider in this case.

B.4.2 $Q^L > 0$

Consider now the case in which Q_L is positive and $\widehat{Q}$ is negative. From previous analysis, this occurs if $a_{T-1} \geq H_{T-1} + \delta\beta A/2 - \delta w$. In this case, three different equilibrium candidates must be considered. Moreover let us distinguish two sub-cases, $\delta\beta\frac{A}{2} - \delta w \geq 0$ and $\delta\beta\frac{A}{2} - \delta w < 0$.

In the case where $\delta\beta\frac{A}{2} - \delta w < 0$, storing is never profitable for speculators: the maximal return net of storage cost they can obtain in the last period is negative. Consequently a stock-out in period $T-1$ always occurs when $\delta\beta\frac{A}{2} - \delta w < 0$, and depending on the sign of $a_{T-1} - H_{T-1}$ the price and the quantity produced in period $T-1$ are positive or nil, and the equilibrium output is equal to

$$q_C = \max\left\{\frac{1}{n}(1-\beta)(a_{T-1} - H_{T-1}), 0\right\} \tag{82}$$

Note that the NP equilibrium in which no production takes place can occur if simultaneously the return of speculation is negative, $\delta\beta\frac{A}{2} - \delta w < 0$, and the initial inventories are too large compared to potential demand, $a_{T-1} - H_{T-1} < 0$.

In the case where $\delta\beta\frac{A}{2} - \delta w \geq 0$, the marginal profit may jump up or down at an aggregate output level equal to Q^L. However as we are working under the assumption that $a_{T-1} - H_{T-1} - \delta\beta\frac{A}{2} + \delta w \geq 0$, the NP equilibrium cannot be an outcome. There are three possible equilibria we must examine: C, CS, and L. Let us start with the L equilibrium, for which each firm' marginal profit is downward jumping.

L **equilibrium.** A limit equilibrium L exists in period $T-1$ when the marginal profit jumps down at the individual output

$$q_L = \frac{Q^L_{T-1}}{n} = \frac{a_{T-1} - H_{T-1} - \delta\beta A/2 + \delta w}{n}, \tag{83}$$

that is when

$$\begin{aligned} &\Pi_q^C(q_{T-1}^L,(n-1)q_{T-1}^L) \geq 0 \\ \Leftrightarrow \quad &a_{T-1} - H_{T-1} \leq \delta \frac{A}{2} - \delta \frac{w}{\beta} \end{aligned} \tag{84}$$

and

$$\begin{aligned} &\Pi_q^{CS}(q_{T-1}^L,(n-1)q_{T-1}^L) \leq 0 \\ \Leftrightarrow \quad &a_{T-1} - H_{T-1} \geq \frac{\delta(1+c)+n+c}{\delta\beta + c(1+\delta\beta)} \left(\delta\beta\frac{A}{2} - \delta w \right) - \frac{\delta\gamma n A}{\delta\beta + c(1+\delta\beta)} \end{aligned} \tag{85}$$

These two conditions are the exact conditions under which an L equilibrium exists, and as the marginal profit is piece-wise decreasing no individual deviation from L exists when these conditions are satisfied.

C **equilibrium.** A C equilibrium will exist if

$$\max\{\Pi_q^C(Q_{T-1}^L-(n-1)q_{T-1}^C,(n-1)q_{T-1}^C), \Pi_q^{CS}(Q_{T-1}^L-(n-1)q_{T-1}^C,(n-1)q_{T-1}^C)\} \leq 0 \tag{86}$$

These conditions can be expressed as a function of the model parameters only, using the expressions of the marginal profits and of the quantities:

$$\begin{aligned} &\Pi_q^C(Q^L-(n-1)q_C,(n-1)q_C) \leq 0 \\ \Leftrightarrow \quad &a_{T-1} - H_{T-1} \geq \frac{(2+c)n(\delta\beta A/2 - \delta w)}{(1+c)(1+(n-1)\beta)} \end{aligned} \tag{87}$$

and

$$\begin{aligned} &\Pi_q^{CS}(Q^L-(n-1)q_C,(n-1)q_C) \leq 0 \\ \Leftrightarrow \quad &a_{T-1} - H_{T-1} \geq \frac{(\delta\beta + (1+\delta\beta)(1+c))n(\delta\beta\frac{A}{2} - \delta w) - \delta n \gamma A}{((1+\delta\beta)(1+c)-1)\,(1+(n-1)\beta)} \end{aligned} \tag{88}$$

If marginal profit is upward jumping at $Q^L-(n-1)q_{T-1}^C$ we need to check that there is no profitable deviation in which a producer triggers speculative storage. We show here how this deviation is computed. Denote $q_d^{CS}(q_C)$ the solution in q to $\Pi_q^{CS}(q,(n-1)q_{T-1}^C)=0$. Following the proof of Proposition 5, marginal profit can be expressed as a function of the level of inventories $H_d^{CS}(q_C)$, and the individual quantity $q_d^{CS}(q_C)$, as:

$$\delta\beta\frac{A}{2}\left(1-\frac{H_d^{CS}(q_C)}{A}\right)^2-\delta w-\left(1+c-\frac{1}{1+\delta\beta\left(1-\frac{H_d^{CS}(q_C)}{A}\right)}\right)q_d^{CS}(q_C)$$
$$-\frac{2\gamma\delta\beta\frac{A}{2}\left(1-\frac{H_d^{CS}(q_C)}{A}\right)^2}{\beta\left(1+\delta\beta\left(1-\frac{H_d^{CS}(q_C)}{A}\right)\right)}=0. \tag{89}$$

Since $Q_d^{CS}(q_C)=q_d^{CS}(q_C)+(n-1)q_C$ must satisfy the arbitrage equation of speculators which links the aggregate quantity to $H_d^{CS}(q_C)$, the deviation quantity must be such that

$$\begin{aligned}q_d^{CS}(q_C)=&a_{T-1}-H_{T-1}+\delta w+A-A\left(1-\frac{H_d^{CS}(q_C)}{A}\right)-\delta\beta\frac{A}{2}\left(1-\frac{H_d^{CS}(q_C)}{A}\right)^2-(n-1)q_C\\ =&\frac{n-(n-1)(1-\beta)}{n}(a_{T-1}-H_{T-1})+\delta w+A-A\left(1-\frac{H_d^{CS}(q_C)}{A}\right)\\ &-\delta\beta\frac{A}{2}\left(1-\frac{H_d^{CS}(q_C)}{A}\right)^2\end{aligned} \tag{90}$$

This expression can be plugged into marginal profit to give a third degree polynomial into $1-\frac{H_d^{CS}(q_C)}{A}=Z_d^{CS}(q_C)$, with coefficients $K_1^d(q_C)$, $K_1^d(q_C)$, $K_3^d(q_C)$ and $K_4^d(q_C)$ to replace the coefficients K_1, K_2, K_3 and K_4 defined in Proposition 5. This polynomial can be solved using Definition 1 and expressions E and F from Proposition 5. Existence of a C equilibrium requires that producing $q_d^{CS}(q_C)$ results in lower profit that is obtained in the C equilibrium.

CS **equilibrium.** A speculation equilibrium CS exists in period $T-1$ when $\Pi_q^{CS}(q_{CS},(n-1)q_{CS})=0$, and when no individual deviation to an aggregate output level implying no inventories to be carried over exists. A sufficient condition for this is

$$\min\{\Pi_q^C(Q^L-(n-1)q_{CS},(n-1)q_{CS}),\Pi_q^{CS}(Q^L-(n-1)q_{CS},(n-1)q_{CS})\}\geq 0 \tag{91}$$

that is:

$$\begin{aligned}&\Pi_q^C(Q^L-(n-1)q_{CS},(n-1)q_{CS})\leq 0\\ \Leftrightarrow\quad & a_{T-1}-H_{T-1}\leq\frac{n(2+c)}{1+c}(\delta\beta A/2-\delta w)+(n-1)\left(\delta w+A\right.\\ &\left.-A\left(-\frac{K_2}{3K_1}+Z^*\right)-\delta\beta\frac{A}{2}\left(-\frac{K_2}{3K_1}+Z^*\right)^2\right)\end{aligned} \tag{92}$$

and

$$\begin{aligned}
&\Pi_q^{CS}(Q^L-(n-1)q_{CS},(n-1)q_{CS})\geq 0\\
\Leftrightarrow\quad & a_{T-1}-H_{T-1}\leq \frac{(n+c+\delta(1+c))(\delta\beta\frac{A}{2}-\delta w)-\delta\gamma nA}{(1+c)(1+\delta\beta)-1}\\
&+\frac{n-1}{n}\left(\delta w+A-A\left(-\frac{K_2}{3K_1}+Z^*\right)-\delta\beta\frac{A}{2}\left(-\frac{K_2}{3K_1}+Z^*\right)^2\right) \qquad (93)
\end{aligned}$$

where Z^* is the solution given in Definition 1.

If marginal profit is upward jumping at $(n-1)q_{T-1}^{CS}-Q^L$ we need to check that there is no profitable deviation in which a producer triggers a stockout. We show here how this deviation is computed. The deviation $q_d^C(q_{CS})$ is the solution in q to $\Pi_q^C(q,(n-1)q_{CS})=0$, which is equal to

$$q_d^C(q_{CS})=\frac{a_{T-1}-H_{T-1}}{2+c}-\frac{n-1}{2+c}q_{CS}, \qquad (94)$$

where q_{CS} has been obtained in proposition 5. Existence of a CS equilibrium requires profit under this deviation be lower than that in the CS equilibrium.

B.4.3 $\widehat{Q}>0$

The case in which Q_L is negative and $\widehat{Q}$ is positive occurs if $a_{T-1}\leq(1-H_{T-1}/A)^2\,\delta\beta A/2-\delta w$. In this case, three different equilibrium candidate must be considered: S, CS, and NP equilibrium. Let us start with the latter.

NP **equilibrium.** An equilibrium involving no production NP exists in period $T-1$ when the marginal profit is negative $\Pi_q^S(0,0)\leq 0$:

$$\Pi_q^S(0,0)=\delta(\beta-2\gamma)\frac{A}{2}\left(1-\frac{H_{T-1}}{A}\right)^2-\delta w\leq 0 \qquad (95)$$

Note that when this condition holds, we can also check that $\Pi_q^{CS}(0,\widehat{Q}_{T-1})\leq 0$, which guarantees that there is not a CS equilibrium:

$$\Pi_q^{CS}(0,\widehat{Q})=\delta\beta\frac{A}{2}\left(1-\frac{H_{T-1}+\widehat{Q}}{A}\right)^2-\delta w-\frac{2\delta\gamma\frac{A}{2}\left(1-\frac{H_{T-1}+\widehat{Q}}{A}\right)}{1+\delta\beta\left(1-\frac{H_{T-1}+\widehat{Q}}{A}\right)}\leq 0 \qquad (96)$$

S **equilibrium.** An S equilibrium exists in period $T-1$ when $\Pi_q^S(q_S,(n-1)q_S)=0$. As the marginal profit is upward jumping at $\widehat{Q}_{T-1}$, a sufficient condition for this equilibrium to exist is

$$
\begin{aligned}
&\Pi_q^{CS}(\widehat{Q}-(n-1)q_S,(n-1)q_S)\leq 0\\
\Leftrightarrow\quad & \delta\beta\frac{A}{2}-\delta w-\left(1+c-\frac{1}{1+\delta\beta}\right)\left(A\left(1-\sqrt{\frac{a_{T-1}+\delta w}{\delta\beta A/2}}\right)-H_{T-1}\right.\\
&\left.-\frac{n-1}{n}\frac{\delta\left(\beta-2\gamma+\frac{\beta}{n}\right)(A-H_{T-1})+\frac{c}{n}A-\sqrt{\Delta}}{\delta\left(\beta-2\gamma+2\frac{\beta}{n}\right)}\right)-\frac{\delta\gamma A}{1+\delta\beta}\leq 0 \qquad (97)
\end{aligned}
$$

where Δ is determined in Proposition 2.

If (97) fails, we must check that no profitable deviation exists that involves a firm inducing sales to consumers. The deviation $q_d^{CS}(q_S)$ can be determined in the same way as was $q_d^{CS}(q_C)$. It is the solution in q to $\Pi_q^{CS}(q,(n-1)q_S)=0$, and is obtained by linking the aggregate quantity $q_d^{CS}(q_S)+(n-1)q_S$ to $H_d^{CS}(q_S)$ through the arbitrage equation of speculators (98) which defines the quantity as a function of speculative inventories:

$$
q_d^{CS}(q_S)+(n-1)q_S=a_{T-1}-H_{T-1}+\delta w+A-A\left(1-\frac{H_d^{CS}(q_S)}{A}\right)-\delta\beta\frac{A}{2}\left(1-\frac{H_d^{CS}(q_S)}{A}\right)^2. \qquad (98)
$$

Using the same method as in the proof of Proposition 5, marginal profit evaluated at this quantity results in a third degree polynomial in $Z\equiv 1-\frac{H_d^{CS}(q_S)}{A}$, the solution of which is used to determine the optimal deviation quantity. The S equilibrium exists if such a deviation yields lower profit than is obtained in the S equilibrium.

***CS* equilibrium.** A CS equilibrium exists in period $T-1$ when $\Pi_q^{CS}(q_{CS},(n-1)q_{CS})=0$ and when no profitable individual deviation exists to an aggregate output level which leads to an exclusion of consumers. Marginal profit is upward jumping at $\widehat{Q}$, so a sufficient condition for this equilibrium to exist is

$$
\begin{aligned}
&\Pi_q^{S}(\widehat{Q}-(n-1)q_{CS},(n-1)q_{CS})\geq 0\\
\Leftrightarrow&\delta(\beta-2\gamma)\frac{A}{2}\frac{a_{T-1}+\delta w}{\delta\beta A/2}-\delta w-\left(\delta\beta\sqrt{\frac{a_{T-1}+\delta w}{\delta\beta A/2}}+c\right)\left(A\left(1-\sqrt{\frac{a_{T-1}+\delta w}{\delta\beta A/2}}\right)-H_{T-1}\right.\\
&\left.-\frac{n-1}{n}\left(a_{T-1}-H_{T-1}+\delta w+A-A\left(Z^*-\frac{K_2}{3K_1}\right)-\delta\beta\frac{A}{2}\left(Z^*-\frac{K_2}{3K_1}\right)^2\right)\right)\geq 0
\end{aligned} \qquad (99)
$$

To check that no profitable deviation exists in which a producer causes consumers to cease buying the product, we compute $q_d^S(q_{CS})$, the solution to $\Pi_q^S(q_d^S,(n-1)q_{CS})=0$. This deviation quantity solves

$$H_d^S(q_{CS}) = H_{T-1} + (n-1)q_{CS} + q_d^S(q_{CS}), \tag{100}$$

or

$$q_d^S(q_{CS}) = A - A\left(1 - \frac{H_d^S(q_{CS})}{A}\right) - H_{T-1} - (n-1)q_{CS}. \tag{101}$$

Using the same method as in the proof of Proposition 2, marginal profit evaluated at this quantity results in a second degree polynomial in $Z \equiv 1 - \frac{H_d^{CS}(q_S)}{A}$, the solution of which determines the optimal deviation quantity. The CS equilibrium exists if such a deviation does not yield higher profits than obtains in the CS equilibrium.

C Numerical Solution Method

In this appendix we provide some addition details about the numerical method used to compute the equilibrium for periods prior to $T-1$.

C.1 Algorithm

We describe here the algorithm for the solution of the T-period model, which is computed with Python code that utilizes numerical routines from NumPy and SciPy[24] (available from the authors on request). The solution is computed recursively beginning from the analytic period T solution described in the paper:

1. Starting value: $\rho_{T-1}(H)$ is given by (15) and $\nu_{T-1}(H)$ by integrating (14).

2. For periods $T-1$ to $T-k$, $\rho_{T-1}(H)$ and $\nu_{T-1}(H)$ are determined by fitting cubic splines to $p_t^e(H_{t+1})$ and $\tilde{V}_t(H_{t+1})$ as follows:

   – Generate expected price and value for each element of a pre-determined vector of next-period stocks at which to perform the approximation,[25] $\bar{H}_k = (0, \bar{H}_k^1, \bar{H}_k^2, ..., \bar{H}_k^m)$.

   – The expected price and value are computed by numerical integration of the Nash equilibrium price and value function in period $T-k+1$ for each $\bar{H}_k^j$. This integration is performed by approximating the uniform distribution for a_t on a uniform grid over $[0, A]$ with 300 values, each with probability 1/300. The expected values for each $\bar{H}_k^j$ are collected in the the vectors $\tilde{p} = (p^0, ..., p^m)$ and $\tilde{V} = (V^0, ..., V^m)$.

[24]Jones et al. (2001–).

[25]The vector of interpolation nodes, $\bar{H}_k$ is allowed to vary with k in order to ensure that the maximal level of stocks at which we perform the interpolation, $\bar{H}_k^m$, is large enough to cause $\rho_{T-k}(\bar{H}_k^m) = 0$. By doing this, we ensure that equilibrium inventories remain within the approximation interval.

- The SciPy routine `interpolate.UnivariateSpline` is used to fit cubic splines to the $(\bar{H}, \tilde{p})$ and $(\bar{H}, \tilde{V})$ data, resulting in the approximations $\rho_{T-k}()$ and $\nu_{T-k}()$.

3. Nash equilibrium calculation: The period t Nash equilibrium is computed using the approximations p^e_{t+1} and $\tilde{V}_{t+1}$ which were computed in the previous step. In commuting this equilibrium numerical routines are required to find $\widetilde{X}_k$, $\widehat{Q}_k$, and q^{CS}_k. We use the SciPy routine `optimize.brentq` for these calculations. An equilibrium selection rule is required for those situations in which multiple equilibria occur. We impose that the C equilibrium is played when both C and CS equilibria exist, and the CS equilibrium is played when both the S and CS equilibria exist. This rule minimizes the amount of storage that occurs. For any (a, H_k) the equilibrium is computed as follows
   - Compute Q^L_k and $\widehat{Q}_k$.
   - If $Q^L_k < 0$ and $\widehat{Q}_k < 0$ the equilibrium is either CS or NP.
   - If Q^L_k the equilibrium is C, L, or CS.
   - If $\widehat{Q}_k > 0$, the equilibrium is S, CS, or NP.
   - In each case, when marginal profit is upward-jumping check that no profitable deviation exists: e.g. for an equilibrium candidate C, if marginal profit is positive at the level of output that induces an aggregate quantity of Q^L_k $(\Pi^{CS}_q(Q^L_k - (n-1)q^C_k, (n-1)q^C_k) > 0)$, compute the optimal deviation for a firm $q^{d/CS} > Q^L_k - (n-1)q^C_k$. If profit from the deviation exceeds that in the C equilibrium candidate, the C equilibrium is ruled out.

To illustrate that the expected price function has "stabilized" by period $T-10$, we plot $\rho_t(H), t = T-10, ..., T$ in Figure 5 for the case $n = 3$. Both the level and slope of $\rho_t(H)$ change very little from $T-10$ to $T-9$, particularly for $H_t = 0$, the case of interest in Figure 4. We plot the expected value function for each period in Figure 6 which shows that by $T-10$, it is increasing in a parallel fashion. The slope of the expected value function is essentially stabilized at $H_t = 0$ by $T-10$.

C.2 Accuracy

We now provide some evidence of the accuracy of our numerical solution. We do this by first examining how accurate the method is at computing the equilibrium in $T-1$ which we can compare with the analytic solution. Second, we examine the approximation residuals for the $T-10$ solution.

In order to examine the accuracy of our method at replicating the period $T-1$ solution, we can compute the interpolated expected price and value functions, $\rho_{T-1}(H_T)$ and $\nu_{T-1}(H_T)$ and compare them with the analytic solutions give by (15) and the integration of (14). For

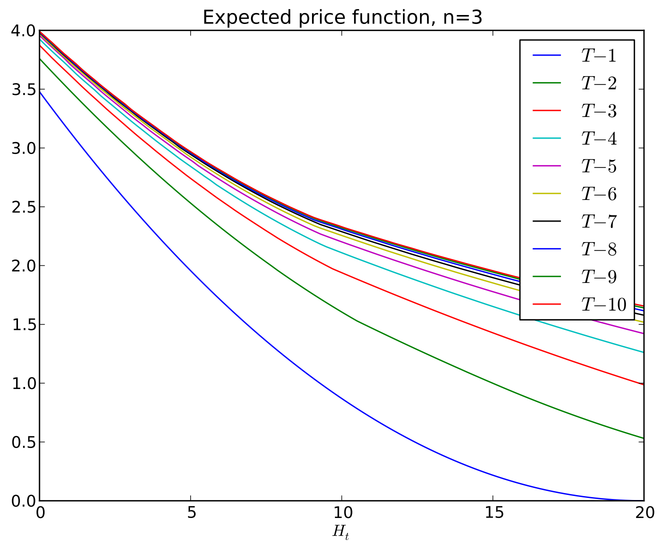

Figure 5: Expected price function, $\rho_t(H), t = T - 10, ..., T$.

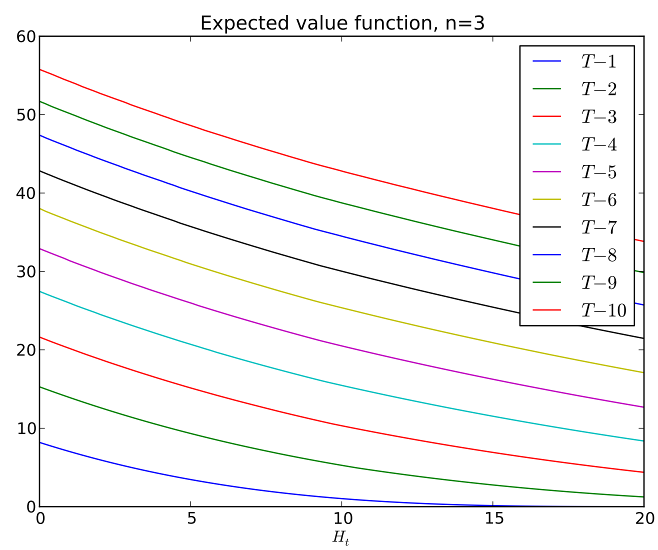

Figure 6: Expected value function, $\nu_t(H), t = T - 10, ..., T$.

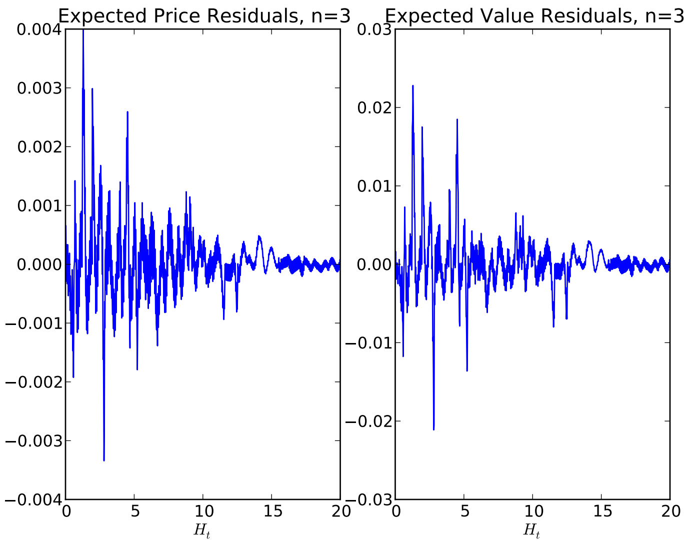

Figure 7: Period $T-10$ approximation residuals for expected price function (left) and expected value function (right).

the parameters given in the paper and $H^m = A = 20$, the difference between the interpolated expected price and value functions an the actual is of the order 10^{-15} for any order of approximation, m, which is not surprising since the expected period T price and value are quadratic functions of H_T. Consequently, the solution using the interpolated approach yields the same results as reported in Figure 1.

To judge the accuracy of the approximation for period prior to $T-1$, we examine the approximation residuals which are computed by comparing the interpolated expected price function, $\rho_t(H)$ to the "actual" expected price on a finer grid than was used to generate the approximation. The "actual" expected price is computed by integrating $p_{t+1}(a_{t+1}, H)$ over a_{t+1} given $\rho_{t+1}(H)$. We use a grid five time finer that the one used to compute Figure 4 and plot the residuals in Figure 7. We were unable to get higher accuracy with this algorithm which seems to be related to difficulty with numerical integration of the price and value functions. Since equilibrium output is discontinuous at points where the type of equilibrium changes when marginal profit is upward jumping, equilibrium price and profit are also discontinuous in a_t and H_t.